Unit 2 Overview

It may surprise you to know that we have now covered all of the basics of physics that we will need for our first three units: position, velocity, acceleration, forces, and Newton's Laws. Moreover, we have already introduced the major ways that we will represent these ideas: words, graphs, simulations, and equations. In this unit, we will be expanding those ideas to more interesting cases where there is more than one force acting at a time and in more than one direction. The only new fundamental principle in this unit is that the various directions of motion, up/down, left/right, forward/backward are independent of each other; A force in the left/right direction does not change the velocity in the up/down or forward/backward directions.

What will we do in this unit then? In order to understand motion in two directions we need a new mathematical tool: vectors. This will be the basics of the first chapter. The second chapter will introduce all the different kinds of forces. In our everyday experience, we see an almost uncountable multitude of different types of forces: pulls of ropes, pushes of hands, springs, gravity, etc. However, these different kinds of forces can actually be grouped into four simple categories. Moreover, at a fundamental level all these different types of forces are different manifestations of only four different fundamental forces. These categorizations of forces will be the subject of the second chapter.

The goal of your prep is for you to develop a basic familiarity with the mechanics of vectors and these different categorizations of forces. In class, we will explore why vectors work the way they do and get practice applying them to situations. Don't worry, therefore, if you feel you don't fully grasp why you are doing the vector tools you are doing right now. Just get the mechanics, we will explore why and how in class. We will then combine vectors, our representations from unit I, and the different kinds of forces to explore some problems that you hopefully find interesting.

# Introduction to Two-Dimensional Kinematics
class="introduction"

Everyday motion that we experience is, thankfully, rarely as tortuous as a rollercoaster ride like this—the Dragon Khan in Spain's Universal Port Aventura Amusement Park. However, most motion is in curved, rather than straight-line, paths. Motion along a curved path is two- or three-dimensional motion, and can be described in a similar fashion to one-dimensional motion. (credit: Boris23/Wikimedia Commons)

The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics. Motion not confined to a plane, such as a car following a winding mountain road, is described by three-dimensional kinematics. Both two- and three-dimensional kinematics are simple extensions of the one-dimensional kinematics developed for straight-line motion in the previous chapter. This simple extension will allow us to apply physics to many more situations, and it will also yield unexpected insights about nature.

Kinematics in Two Dimensions-An introduction

- A vector is a quantity with a magnitude and direction
- Converting between magnitude/direction and the component form for any vector. This ties into the Pythagorean Theorem

Exercise:

UMASS AMHERST Instructor's Notes

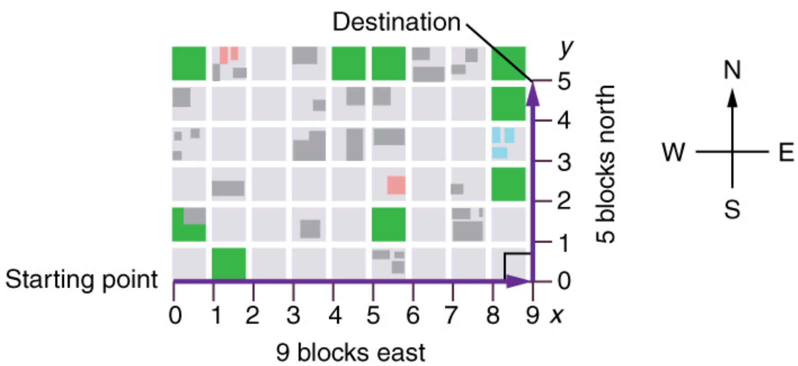
Problem:
Your Quiz would Cover

- A vector is a quantity with a magnitude and direction
- Converting between magnitude/direction and the component form for any vector. This ties into the Pythagorean Theorem

Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. (credit: Margaret W. Carruthers)

Two-Dimensional Motion: Walking in a City

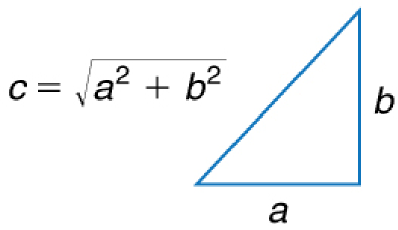
Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in [link].



A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem, $a^2 + b^2 = c^2$, can be used to find the straight-line distance.



The Pythagorean theorem relates the length of the legs of a right triangle, labeled a and b, with the hypotenuse, labeled c. The relationship is given by: $a^2+b^2=c^2$. This can be rewritten, solving for c : $c=\sqrt{a^2+b^2}$.

Exercise:

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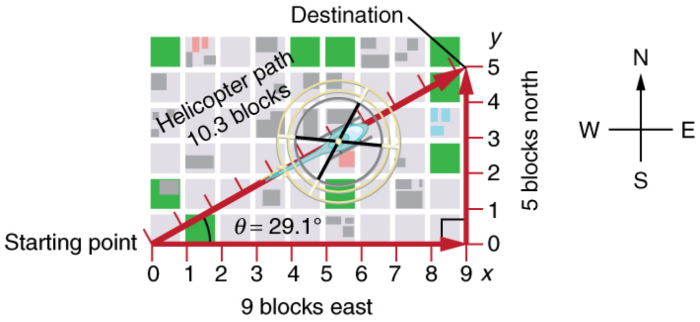
Problem:

We will be using the pythagorean theorem all throughout two-dimensional kinematics, as well as throughout this entire course. If you are uncomfortable or unfamiliar with the Pythagorean Theorem, or even if it's just been a long time since you've used it, please come see your instructor as soon as possible and they will get you up to speed.

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is

$\sqrt{(9\text{ blocks})^2 + (5\text{ blocks})^2} = 10.3\text{ blocks}$, considerably shorter than the 14 blocks you walked. (Note that we are using three significant figures in the answer. Although it appears that “9” and “5” have only one significant digit, they are discrete numbers. In this case “9 blocks” is the same as “9.0 or 9.00 blocks.” We have decided to use three significant figures in the answer in order to show the result more precisely.)



The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.

The fact that the straight-line distance (10.3 blocks) in [link] is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that **vectors** are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector’s magnitude. The arrow’s length is indicated by hash marks in [link] and [link]. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the

vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in [link]. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods.)

The Independence of Perpendicular Motions

Exercise:

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Problem:

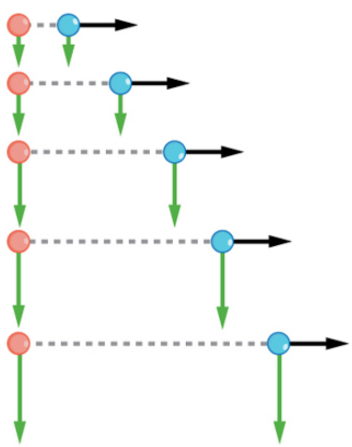
The idea of the independence of perpendicular motion is a fundamental one that you should take some time to think about, and there are some questions about this on the homework.

The person taking the path shown in [link] walks east and then north (two perpendicular directions). How far he or she walks east is only affected by his or her motion eastward. Similarly, how far he or she walks north is only affected by his or her motion northward.

Note:

Independence of Motion
The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let's compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.



This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an

equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

Exercise:

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Problem:

This graphic displays this concept quite nicely; notice how both balls fall downward at the same speed at each point, even though one of the balls has a horizontal velocity. Basically, the velocity of the ball in the x-direction has no effect on the velocity in the y-direction, and vice-versa. This will be an important idea, especially when working with vectors.

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods. We will find such techniques to be useful in many areas of physics.

Note:
PhET Explorations: Ladybug Motion 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

Ladybug Motion 2D

Summary

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

Glossary

vector
: a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

Vector Addition and Subtraction: Graphical Methods

- Given two graphical representations of vectors, be able to draw the sum or difference.
- Describe both visually and mathematically what happens when a scalar is multiplied by a vector.
- Convert between magnitude/direction and component form for any vector

Exercise:

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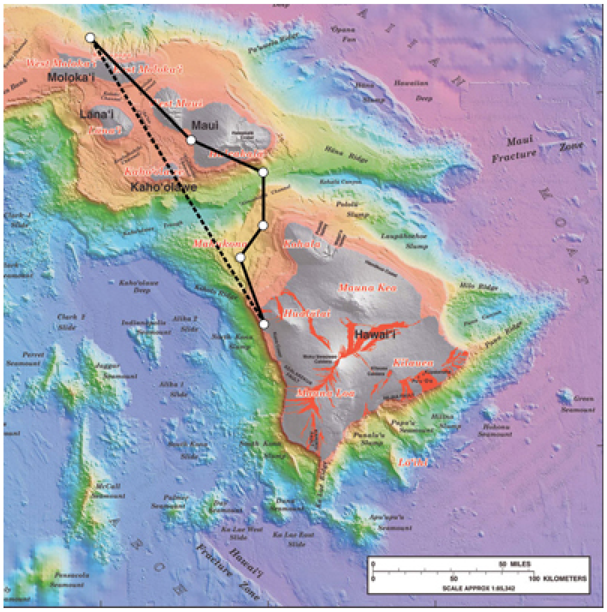
Problem:
Your Quiz would Cover

- Given two graphical representations of vectors, be able to draw the sum or difference. There are some simple procedures to follow. Solidify your understanding of these procedures and we can work on why this makes sense in class
- Describe both visually and mathematically what happens when a scalar is multiplied by a vector. If I give you a vector and a number, you should be able to turn the crank and multiply them mathematically. I am NOT expecting you to be able to do this graphically and will not ask you what it means. Just focus on the mechanics of how to do it.
- Convert between magnitude/direction and component form for any vector



Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai’i to Moloka’i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey. (credit: US Geological Survey)

Vectors in Two Dimensions

A **vector** is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In two dimensions (2-d), however, we specify the direction of a vector relative to some reference frame (i.e.,

coordinate system), using an arrow having length proportional to the vector’s magnitude and pointing in the direction of the vector.

[link] shows such a *graphical representation of a vector*, using as an example the total displacement for the person walking in a city considered in Kinematics in Two Dimensions: An Introduction. We shall use the notation that a boldface symbol, such as $\mathbf{D}$, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction by θ.

Exercise:

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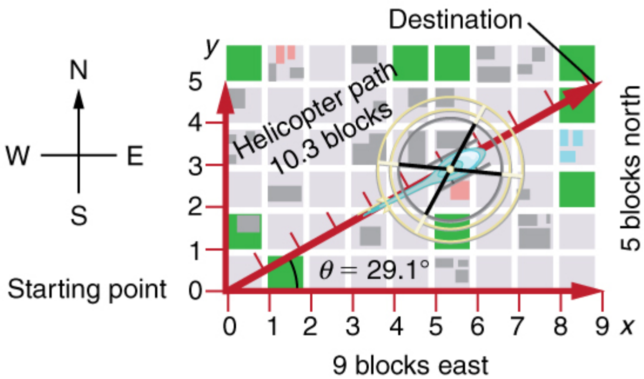
Problem:

There's some notation in the following note that would be useful to pay attention too.

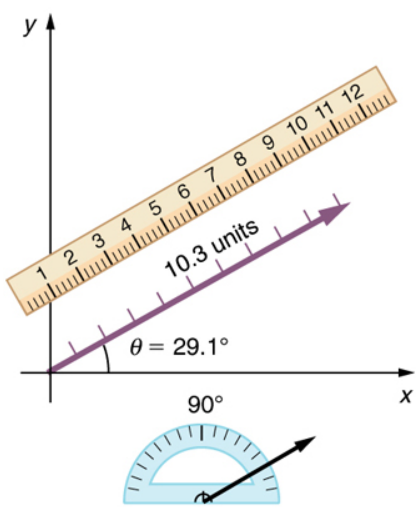
Note:

Vectors in this Text

In this text, we will represent a vector with a boldface variable. For example, we will represent the quantity force with the vector $\mathbf{F}$, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as F, and the direction of the variable will be given by an angle θ.



A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle 29.1° north of east.



To describe the resultant vector for the person walking in a city considered in [link] graphically, draw an arrow to represent the total

displacement vector D. Using a protractor, draw a line at an angle θ relative to the east-west axis. The length D of the arrow is proportional to the vector's magnitude and is measured along the line with a ruler. In this example, the magnitude D of the vector is 10.3 units, and the direction θ is 29.1° north of east.

Exercise:

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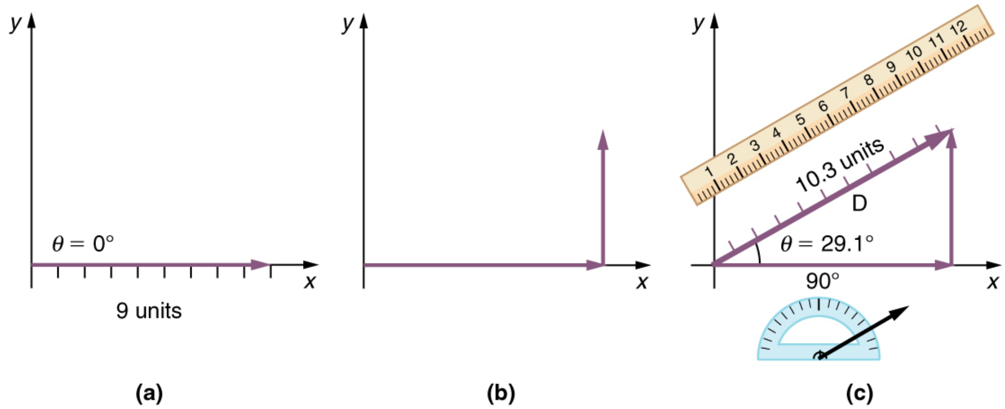
Problem:

Taking some time to understand and practice the head-to-tail method is recommended, you'll notice that there's a series of algorithmic steps, so you just need to learn the process, and it will work for any two vectors.

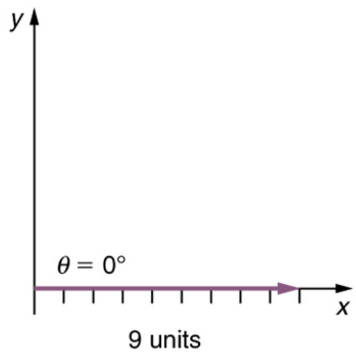
Vector Addition: Head-to-Tail Method

The **head-to-tail method** is a graphical way to add vectors, described in [link] below and in the steps following. The **tail** of the vector is the starting point of the vector, and the **head** (or tip) of a vector is the final, pointed end of the arrow.



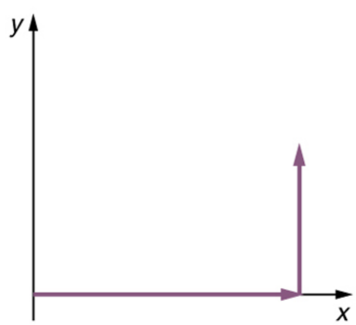
Head-to-Tail Method: The head-to-tail method of graphically adding vectors is illustrated for the two displacements of the person walking in a city considered in [link]. (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector. (c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or **resultant vector** D. The length of the arrow D is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis) θ is measured with a protractor to be 29.1^{o}.

Step 1. *Draw an arrow to represent the first vector (9 blocks to the east) using a ruler and protractor.*



(a)

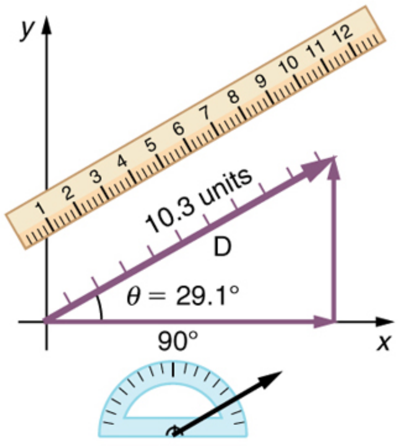
Step 2. Now draw an arrow to represent the second vector (5 blocks to the north). *Place the tail of the second vector at the head of the first vector*.



(b)

Step 3. *If there are more than two vectors, continue this process for each vector to be added. Note that in our example, we have only two vectors, so we have finished placing arrows tip to tail*.

Step 4. *Draw an arrow from the tail of the first vector to the head of the last vector*. This is the **resultant**, or the sum, of the other vectors.



(c)

Step 5. To get the **magnitude** of the resultant, *measure its length with a ruler. (Note that in most calculations, we will use the Pythagorean theorem to determine this length.)*

Step 6. To get the **direction** of the resultant, *measure the angle it makes with the reference frame using a protractor. (Note that in most calculations, we will use trigonometric relationships to determine this angle.)*

The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

Example:
Adding Vectors Graphically Using the Head-to-Tail Method: A Woman Takes a Walk

Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25.0 m in a direction 49.0º north of east. Then, she walks 23.0 m heading 15.0º north of east. Finally, she turns and walks 32.0 m in a direction 68.0° south of east.

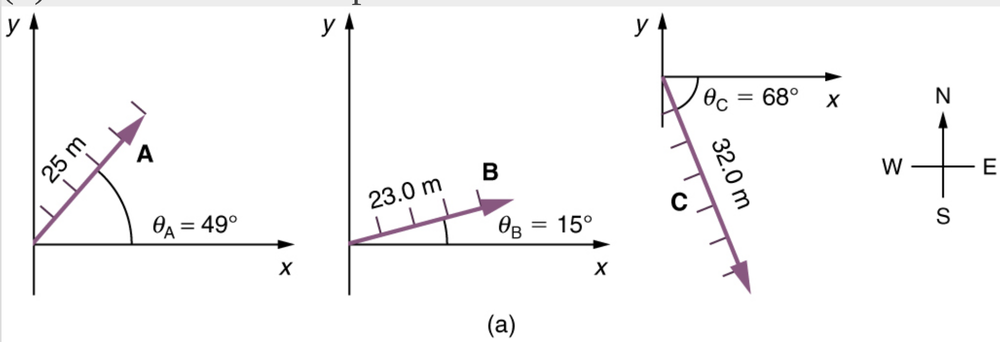
Strategy

Represent each displacement vector graphically with an arrow, labeling the first A, the second B, and the third C, making the lengths proportional to the distance and the directions as specified relative to an east-west line.

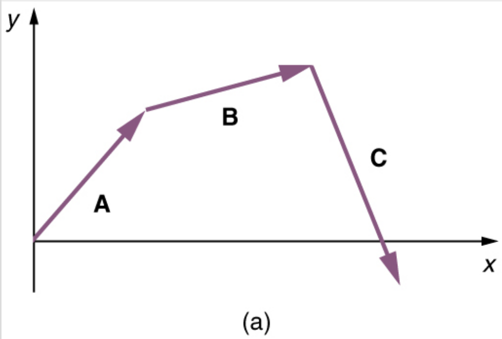
The head-to-tail method outlined above will give a way to determine the magnitude and direction of the resultant displacement, denoted **R**.

Solution

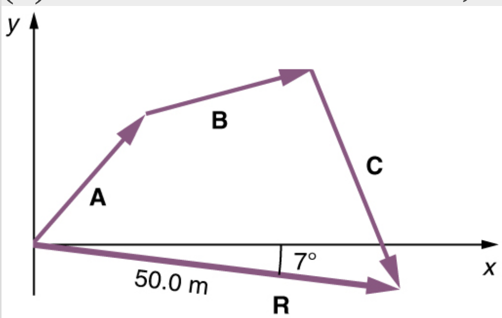
(1) Draw the three displacement vectors.



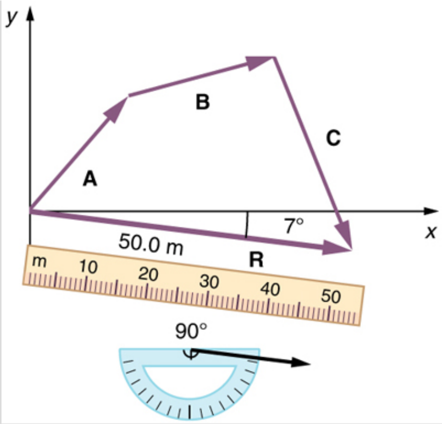
(2) Place the vectors head to tail retaining both their initial magnitude and direction.



(3) Draw the resultant vector, R.



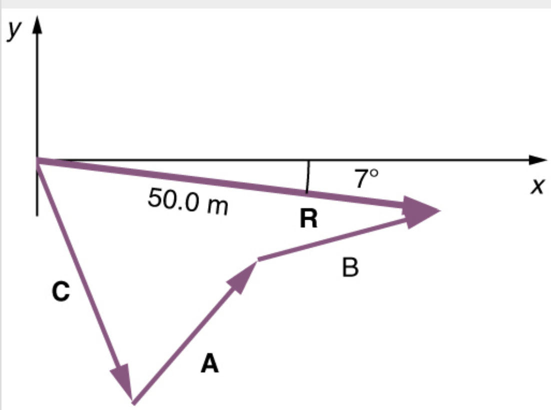
(4) Use a ruler to measure the magnitude of **R**, and a protractor to measure the direction of R. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since the resultant vector is south of the eastward pointing axis, we flip the protractor upside down and measure the angle between the eastward axis and the vector.



In this case, the total displacement $\mathbf{R}$ is seen to have a magnitude of 50.0 m and to lie in a direction 7.0^{o} south of east. By using its magnitude and direction, this vector can be expressed as $R = 50.0\text{ m}$ and $\theta = 7.0^{\text{o}}$ south of east.

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that the resultant is independent of the order in which the vectors are added. Therefore, we could add the vectors in any order as illustrated in [link] and we will still get the same solution.



Here, we see that when the same vectors are added in a different order, the result is the same. This characteristic is true in every case and is an important characteristic of vectors. Vector addition is **commutative**. Vectors can be added in any order.

Equation:

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}.$$

(This is true for the addition of ordinary numbers as well—you get the same result whether you add $\mathbf{2 + 3}$ or $\mathbf{3 + 2}$, for example).

Exercise:

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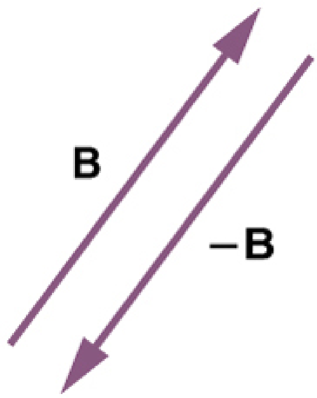
Problem:

Understanding vector subtraction is necessary to understand other physics ideas. For example, acceleration is Δv/Δt, and Δv is v_f - v_i. Velocity is a vector, so you're looking at a vector subtraction whenever you're working with acceleration.

Vector Subtraction

Vector subtraction is a straightforward extension of vector addition. To define subtraction (say we want to subtract $\mathbf{B}$ from $\mathbf{A}$, written $\mathbf{A} – \mathbf{B}$, we must first define what we mean by subtraction. The *negative* of a vector $\mathbf{B}$ is defined to be $–\mathbf{B}$; that is, graphically *the negative of any vector has the same magnitude but the opposite direction*, as shown in [link]. In other words, $\mathbf{B}$ has the same length as $–\mathbf{B}$, but points in the opposite direction. Essentially, we just flip the vector so it points in the opposite direction.



The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So $\mathbf{B}$ is the negative of $-\mathbf{B}$; it has the same length but opposite direction.

The *subtraction* of vector $\mathbf{B}$ from vector $\mathbf{A}$ is then simply defined to be the addition of $-\mathbf{B}$ to $\mathbf{A}$. Note that vector subtraction is the addition of a negative vector. The order of subtraction does not affect the results.

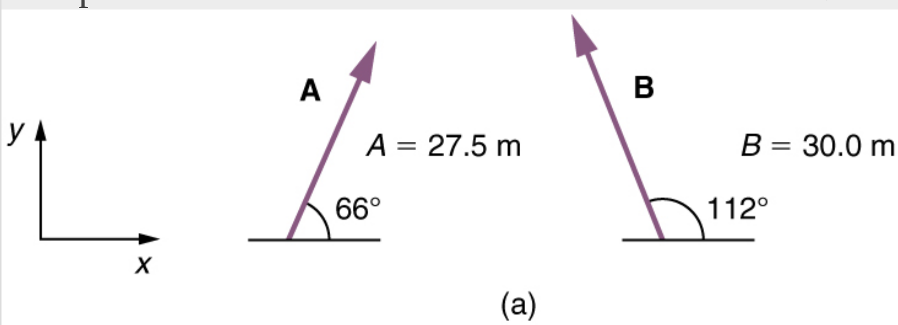
Equation:

$$\mathrm{A} - \mathrm{B} = \mathrm{A} + (-\mathbf{B}).$$

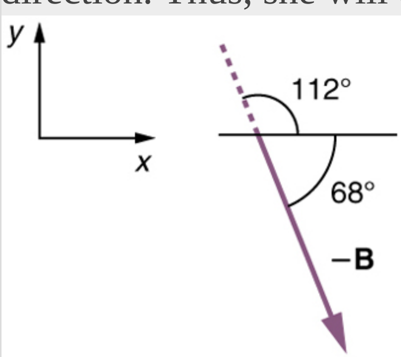
This is analogous to the subtraction of scalars (where, for example, $5 - 2 = 5 + (-2)$). Again, the result is independent of the order in which

the subtraction is made. When vectors are subtracted graphically, the techniques outlined above are used, as the following example illustrates.

Example:
Subtracting Vectors Graphically: A Woman Sailing a Boat
A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction 66.0^{o} north of east from her current location, and then travel 30.0 m in a direction 112^{o} north of east (or 22.0^{o} west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? Compare this location with the location of the dock.



Strategy
We can represent the first leg of the trip with a vector $\mathbf{A}$, and the second leg of the trip with a vector $\mathbf{B}$. The dock is located at a location $\mathbf{A}+\mathbf{B}$. If the woman mistakenly travels in the *opposite* direction for the second leg of the journey, she will travel a distance B (30.0 m) in the direction $180^{\mathrm{o}}-112^{\mathrm{o}}=68^{\mathrm{o}}$ south of east. We represent this as $-\mathbf{B}$, as shown below. The vector $-\mathbf{B}$ has the same magnitude as $\mathbf{B}$ but is in the opposite direction. Thus, she will end up at a location $\mathbf{A}+(-\mathbf{B})$, or $\mathbf{A}-\mathbf{B}$.



We will perform vector addition to compare the location of the dock, $\mathrm{A}+\mathbf{B}$, with the location at which the woman mistakenly arrives, $\mathrm{A}+(-\mathrm{B})$.

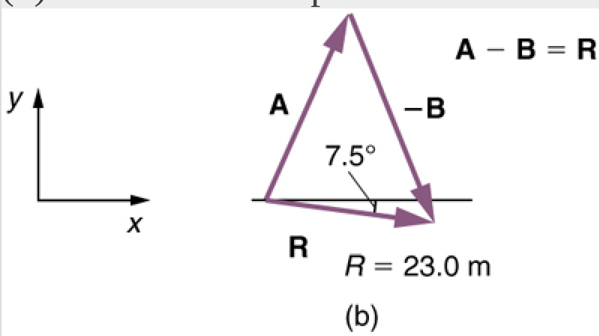
Solution

(1) To determine the location at which the woman arrives by accident, draw vectors $\mathbf{A}$ and $-\mathbf{B}$.

(2) Place the vectors head to tail.

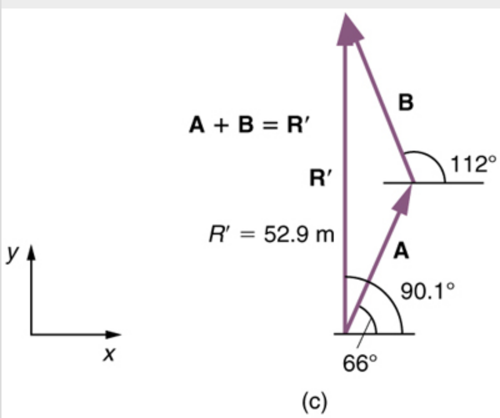
(3) Draw the resultant vector $\mathbf{R}$.

(4) Use a ruler and protractor to measure the magnitude and direction of $\mathbf{R}$.



In this case, $R=23.0\text{ m}$ and $\theta=7.5^{\circ}$ south of east.

(5) To determine the location of the dock, we repeat this method to add vectors $\mathbf{A}$ and $\mathbf{B}$. We obtain the resultant vector $\mathbf{R}\prime$:



In this case $R=52.9\text{ m}$ and $\theta=90.1^{\circ}$ north of east.

We can see that the woman will end up a significant distance from the dock if she travels in the opposite direction for the second leg of the trip.

Discussion

Because subtraction of a vector is the same as addition of a vector with the opposite direction, the graphical method of subtracting vectors works the same as for addition.

Exercise:

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Problem:

If you've taken another physics course, you've probably seen **F** = *m***a**. This equation will play a significant role in this class, and you'll notice that mass is a scalar, and acceleration is a vector, so understanding how scalars and vectors multiply will be important.

Multiplication of Vectors and Scalars

If we decided to walk three times as far on the first leg of the trip considered in the preceding example, then we would walk $3\ \times\ 27.5\ \text{m}$, or 82.5 m, in a direction 66.0^{o} north of east. This is an example of multiplying a vector by a positive **scalar**. Notice that the magnitude changes, but the direction stays the same.

If the scalar is negative, then multiplying a vector by it changes the vector's magnitude and gives the new vector the *opposite* direction. For example, if you multiply by –2, the magnitude doubles but the direction changes. We can summarize these rules in the following way: When vector $\mathbf{A}$ is multiplied by a scalar c,

- the magnitude of the vector becomes the absolute value of cA,
- if c is positive, the direction of the vector does not change,
- if c is negative, the direction is reversed.

In our case, $c=3$ and $A=27.5\ \text{m}$. Vectors are multiplied by scalars in many situations. Note that division is the inverse of multiplication. For example, dividing by 2 is the same as multiplying by the value (1/2). The rules for multiplication of vectors by scalars are the same for division; simply treat the divisor as a scalar between 0 and 1.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

When dealing with vectors using analytic methods (which is covered in the next section), you need to break down vectors into essentially x-components and y-components. This next part covers this idea, so try to familiarize yourself with breaking down vectors as you read.

Resolving a Vector into Components

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular **components** of a single vector, for example the *x- and y-*components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a direction 29.0° north of east and want to find out how many blocks east and north had to be walked. This method is called *finding the components (or parts)* of the displacement in the east and north directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. There are many applications in physics where this is a useful thing to do. We will see this soon in Projectile Motion, and much more when we cover **forces** in Dynamics: Newton's Laws of Motion. Most of these involve finding components along perpendicular axes (such as north and east), so that right triangles are involved. The analytical techniques presented in Vector Addition and Subtraction: Analytical Methods are ideal for finding vector components.

Note:

PhET Explorations: Maze Game

Learn about position, velocity, and acceleration in the "Arena of Pain". Use the green arrow to move the ball. Add more walls to the arena to make the game more difficult. Try to make a goal as fast as you can.

Maze Game

Summary

- The **graphical method of adding vectors** $\mathbf{A}$ and $\mathbf{B}$ involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector $\mathbf{R}$ is defined such that $\mathbf{A}+\mathbf{B}=\mathbf{R}$. The magnitude and direction of $\mathbf{R}$ are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector** $\mathbf{B}$ from $\mathbf{A}$ involves adding the opposite of vector $\mathbf{B}$, which is defined as $-\mathbf{B}$. In this case, $\mathbf{A}–\mathbf{B}=\mathbf{A}+(-\mathbf{B})=\mathbf{R}$. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector $\mathbf{R}$.
- Addition of vectors is **commutative** such that $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$.
- The **head-to-tail method** of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector $\mathbf{A}$ is multiplied by a scalar quantity c, the magnitude of the product is given by cA. If c is positive, the direction of the product points in the same direction as $\mathbf{A}$; if c is negative, the direction of the product points in the opposite direction as $\mathbf{A}$.

Conceptual Questions

Exercise:

Problem:

Which of the following is a vector: a person's height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth's population, the acceleration of gravity?

Exercise:

Problem:

Give a specific example of a vector, stating its magnitude, units, and direction.

Exercise:

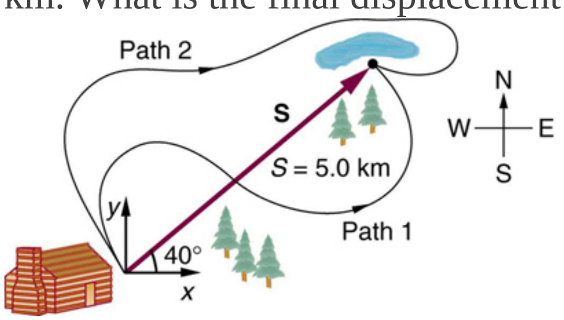
Problem:

What do vectors and scalars have in common? How do they differ?

Exercise:

Problem:

Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is 8.2 km. What is the final displacement of each camper?



Exercise:

Problem:

If an airplane pilot is told to fly 123 km in a straight line to get from San Francisco to Sacramento, explain why he could end up anywhere on the circle shown in [link]. What other information would he need to get to Sacramento?

Exercise:

Problem:

Suppose you take two steps $\mathbf{A}$ and $\mathbf{B}$ (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give zero? Is the maximum distance you can end up from the starting point $\mathbf{A}+\mathbf{B}$ the sum of the lengths of the two steps?

Exercise:

Problem: Explain why it is not possible to add a scalar to a vector.

Exercise:

Problem:

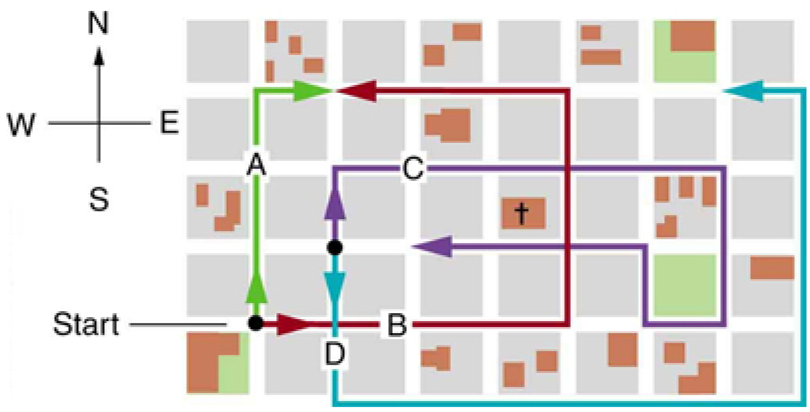
If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

Problems & Exercises

Use graphical methods to solve these problems. You may assume data taken from graphs is accurate to three digits.
Exercise:

Problem:

Find the following for path A in [link]: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.



The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

Solution:

(a) 480 m

(b) 379 m, 18.4º east of north

Exercise:

Problem:

Find the following for path B in [link]: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

Exercise:

Problem:

Find the north and east components of the displacement for the hikers shown in [link].

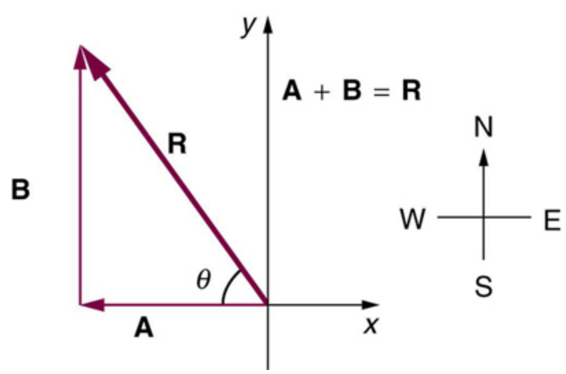
Solution:

north component 3.21 km, east component 3.83 km

Exercise:

Problem:

Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in [link], then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

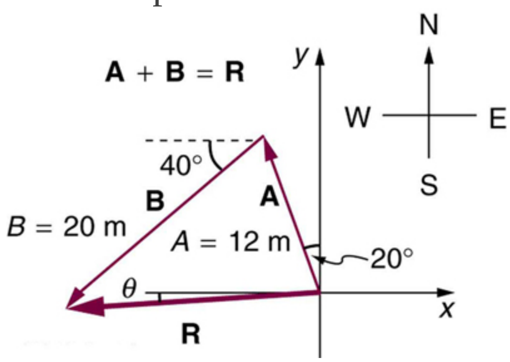


The two displacements $\mathbf{A}$ and $\mathbf{B}$ add to give a total displacement $\mathbf{R}$ having magnitude R and direction θ.

Exercise:

Problem:

Suppose you first walk 12.0 m in a direction 20° west of north and then 20.0 m in a direction 40.0° south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in [link], then this problem finds their sum $\mathrm{R} = \mathrm{A} + \mathrm{B}$.)



Solution:

$19.5\ \mathrm{m}$, 4.65° south of west

Exercise:

Problem:

Repeat the problem above, but reverse the order of the two legs of the walk; show that you get the same final result. That is, you first walk leg $\mathbf{B}$, which is 20.0 m in a direction exactly 40° south of west, and then leg $\mathbf{A}$, which is 12.0 m in a direction exactly 20° west of north. (This problem shows that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.)

Exercise:

Problem:

(a) Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction 40.0^{o} north of east (which is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, to finding $\mathbf{R}\prime = \mathbf{A} - \mathbf{B}$). (b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction 40.0^{o} south of west and then 12.0 m in a direction 20.0^{o} east of south (which is equivalent to subtracting $\mathbf{A}$ from $\mathbf{B}$ —that is, to finding $\mathbf{R}\prime\prime = \mathbf{B} - \mathbf{A} = -\mathbf{R}\prime$). Show that this is the case.

Solution:

(a) $26.6\text{ m}, 65.1^{\mathrm{o}}$ north of east

(b) $26.6\text{ m}, 65.1^{\mathrm{o}}$ south of west

Exercise:

Problem:

Show that the *order* of addition of three vectors does not affect their sum. Show this property by choosing any three vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, all having different lengths and directions. Find the sum $\mathrm{A} + \mathrm{B} + \mathrm{C}$ then find their sum when added in a different order and show the result is the same. (There are five other orders in which $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ can be added; choose only one.)

Exercise:

Problem:

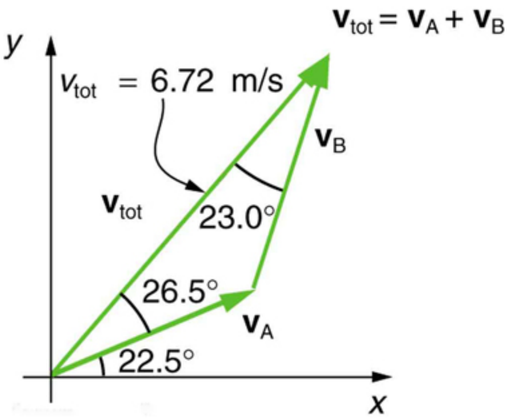
Show that the sum of the vectors discussed in [link] gives the result shown in [link].

Solution:

$52.9\text{ m}, 90.1^{\mathrm{o}}$ with respect to the *x*-axis.

Exercise:

Problem: Find the magnitudes of velocities v_A and v_B in [link]



The two velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ add to give a total $\mathbf{v}_{tot}$.

Exercise:

Problem:

Find the components of v_{tot} along the *x*- and *y*-axes in [link].

Solution:

x-component 4.41 m/s

y-component 5.07 m/s

Exercise:

Problem:

Find the components of v_{tot} along a set of perpendicular axes rotated 30º counterclockwise relative to those in [link].

Glossary

component (of a 2-d vector)
: a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

commutative
: refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum

direction (of a vector)
: the orientation of a vector in space

head (of a vector)
: the end point of a vector; the location of the tip of the vector's arrowhead; also referred to as the "tip"

head-to-tail method
: a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

magnitude (of a vector)
: the length or size of a vector; magnitude is a scalar quantity

resultant
: the sum of two or more vectors

resultant vector
: the vector sum of two or more vectors

scalar
: a quantity with magnitude but no direction

tail
: the start point of a vector; opposite to the head or tip of the arrow

Vector Addition and Subtraction: Analytical Methods

- Adding vectors by components

Exercise:

UMASS AMHERST Instructor's Notes

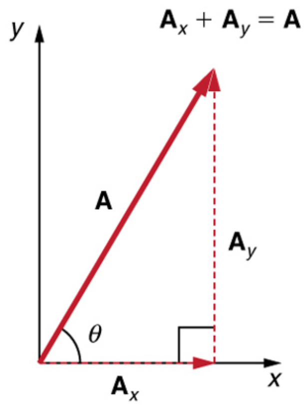
Problem:
Your Quiz would Cover

- Adding vectors by components. Don’t focus too much on what it means to add vectors. Just learn the mechanics of how to do it. We will talk about the meaning in class.

Analytical methods of vector addition and subtraction employ geometry and simple trigonometry rather than the ruler and protractor of graphical methods. Part of the graphical technique is retained, because vectors are still represented by arrows for easy visualization. However, analytical methods are more concise, accurate, and precise than graphical methods, which are limited by the accuracy with which a drawing can be made. Analytical methods are limited only by the accuracy and precision with which physical quantities are known.

Resolving a Vector into Perpendicular Components

Analytical techniques and right triangles go hand-in-hand in physics because (among other things) motions along perpendicular directions are independent. We very often need to separate a vector into perpendicular components. For example, given a vector like $\mathbf{A}$ in [link], we may wish to find which two perpendicular vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, add to produce it.



The vector $\mathbf{A}$, with its tail at the origin of an *x*, *y*-coordinate system, is shown together with its *x*- and *y*-components, $\mathbf{A}_x$ and $\mathbf{A}_y$. These vectors form a right triangle. The analytical relationships among these vectors are summarized below.

$\mathbf{A}_x$ and $\mathbf{A}_y$ are defined to be the components of $\mathbf{A}$ along the *x*- and *y*-axes. The three vectors $\mathbf{A}$, $\mathbf{A}_x$, and $\mathbf{A}_y$ form a right triangle:

Equation:

$$\mathbf{A}_x + \mathrm{A}_y = \mathrm{A}.$$

Note that this relationship between vector components and the resultant vector holds only for vector quantities (which include both magnitude and direction). The relationship does not apply for the magnitudes alone. For example, if $\mathbf{A}_x = 3\text{ m}$ east, $\mathbf{A}_y = 4\text{ m}$ north, and $\mathbf{A} = 5\text{ m}$ north-east, then it is true that the vectors $\mathbf{A}_x + \mathrm{A}_y = \mathrm{A}$. However, it is *not* true that the sum of the magnitudes of the vectors is also equal. That is,
Equation:

$$3\text{ m} + 4\text{ m} \neq 5\text{ m}$$

Thus,
Equation:

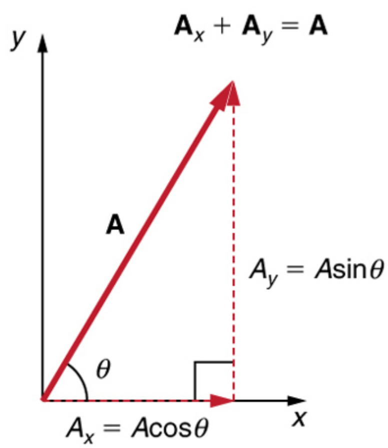
$$A_x + A_y \neq A$$

If the vector $\mathbf{A}$ is known, then its magnitude A (its length) and its angle θ (its direction) are known. To find A_x and A_y, its *x*- and *y*-components, we use the following relationships for a right triangle.
Equation:

$$A_x = A\cos\theta$$

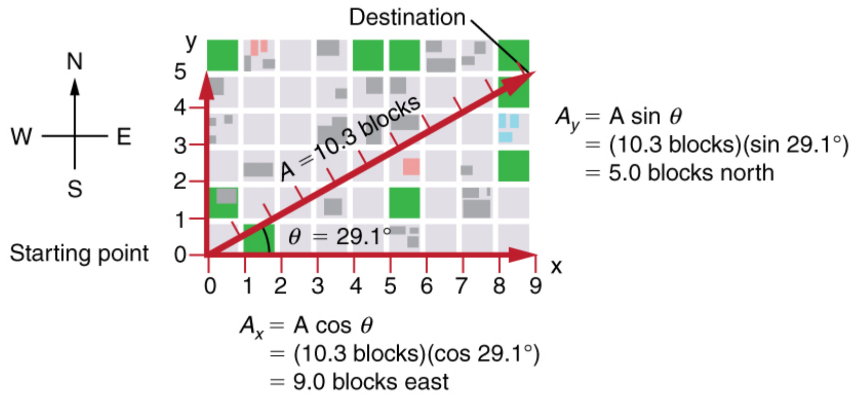
and
Equation:

$$A_y = A\sin\theta.$$



The magnitudes of the vector components $\mathbf{A}_x$ and $\mathbf{A}_y$ can be related to the resultant vector $\mathbf{A}$ and the angle θ with trigonometric identities. Here we see that $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

Suppose, for example, that $\mathbf{A}$ is the vector representing the total displacement of the person walking in a city considered in Kinematics in Two Dimensions: An Introduction and Vector Addition and Subtraction: Graphical Methods.



We can use the relationships $A_x = A \cos \theta$ and $A_y = A \sin \theta$ to determine the magnitude of the horizontal and vertical component vectors in this example.

Then $A = 10.3$ blocks and $\theta = 29.1^\text{o}$, so that

Equation:

$$A_x = A \cos \theta = (10.3 \text{ blocks})(\cos 29.1^\text{o}) = 9.0 \text{ blocks}$$

Equation:

$$A_y = A \sin \theta = (10.3 \text{ blocks})(\sin 29.1^\text{o}) = 5.0 \text{ blocks.}$$

Calculating a Resultant Vector

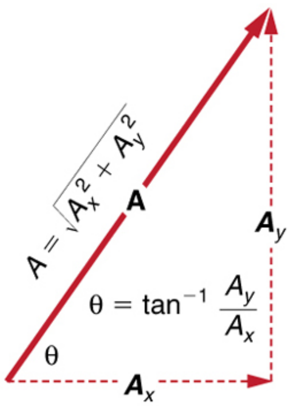
If the perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$ of a vector $\mathbf{A}$ are known, then $\mathbf{A}$ can also be found analytically. To find the magnitude A and direction θ of a vector from its perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$, we use the following relationships:

Equation:

$$A = \sqrt{A_{x^2} + A_{y^2}}$$

Equation:

$$\theta = \tan^{-1}(A_y/A_x).$$



The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components A_x and A_y have been determined.

Note that the equation $A = \sqrt{A_x^2 + A_y^2}$ is just the Pythagorean theorem relating the legs of a right triangle to the length of the hypotenuse. For

example, if A_x and A_y are 9 and 5 blocks, respectively, then $A = \sqrt{9^2+5^2}$=10.3 blocks, again consistent with the example of the person walking in a city. Finally, the direction is $\theta = \tan^{-1}(5/9)$=29.1° , as before.

> **Note:**
> Determining Vectors and Vector Components with Analytical Methods
> Equations $A_x = A \cos \theta$ and $A_y = A \sin \theta$ are used to find the perpendicular components of a vector—that is, to go from A and θ to A_x and A_y. Equations $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}(A_y/A_x)$ are used to find a vector from its perpendicular components—that is, to go from A_x and A_y to A and θ. Both processes are crucial to analytical methods of vector addition and subtraction.

Exercise:

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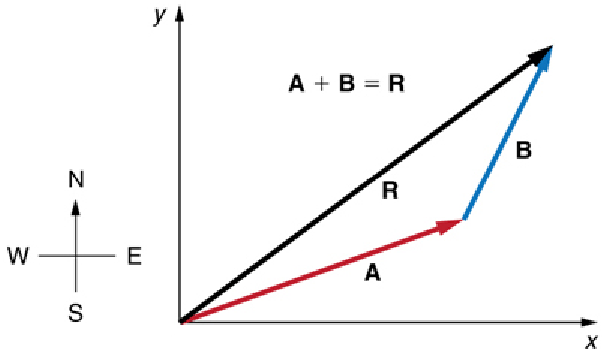
Problem:

Now that you know how to break down vectors into components, here's a procedure to adding vectors analytically. There's some trigonometry involved, so, again, if you're not familiar or comfortable with trigonometry, come see your instructor. You should be familiar with both methods. you should be able to add two vectors given their x and y components, and you should be able to draw the resulting vector of two added vectors. Also, we will go over how to use these to solve problems, so focus primarily on the methods of adding vectors.

Adding Vectors Using Analytical Methods

To see how to add vectors using perpendicular components, consider [link], in which the vectors $\mathbf{A}$ and $\mathbf{B}$ are added to produce the resultant $\mathbf{R}$.

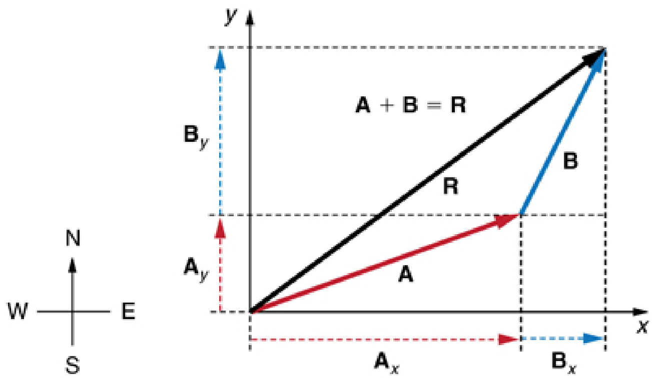


Vectors $\mathbf{A}$ and $\mathbf{B}$ are two legs of a walk, and $\mathbf{R}$ is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of $\mathbf{R}$.

If $\mathbf{A}$ and $\mathbf{B}$ represent two legs of a walk (two displacements), then $\mathbf{R}$ is the total displacement. The person taking the walk ends up at the tip of $\mathbf{R}$. There are many ways to arrive at the same point. In particular, the person could have walked first in the *x*-direction and then in the *y*-direction. Those paths are the *x*- and *y*-components of the resultant, $\mathbf{R}_x$ and $\mathbf{R}_y$. If we know $\mathbf{R}_x$ and $\mathbf{R}_y$, we can find R and θ using the equations $A = \sqrt{{A_x}^2 + {A_y}^2}$ and $\theta = \tan^{-1}(A_y/A_x)$. When you use the analytical method of vector addition, you can determine the components or the magnitude and direction of a vector.

Step 1. *Identify the x- and y-axes that will be used in the problem. Then, find the components of each vector to be added along the chosen perpendicular axes*. Use the equations $A_x = A \cos \theta$ and $A_y = A \sin \theta$ to find the components. In [link], these components are A_x, A_y, B_x, and B_y.

The angles that vectors $\mathbf{A}$ and $\mathbf{B}$ make with the *x*-axis are θ_A and θ_B, respectively.



To add vectors $\mathbf{A}$ and $\mathbf{B}$, first determine the horizontal and vertical components of each vector. These are the dotted vectors $\mathbf{A}_x$, $\mathbf{A}_y$, $\mathbf{B}_x$ and $\mathbf{B}_y$ shown in the image.

Step 2. *Find the components of the resultant along each axis by adding the components of the individual vectors along that axis.* That is, as shown in [link],

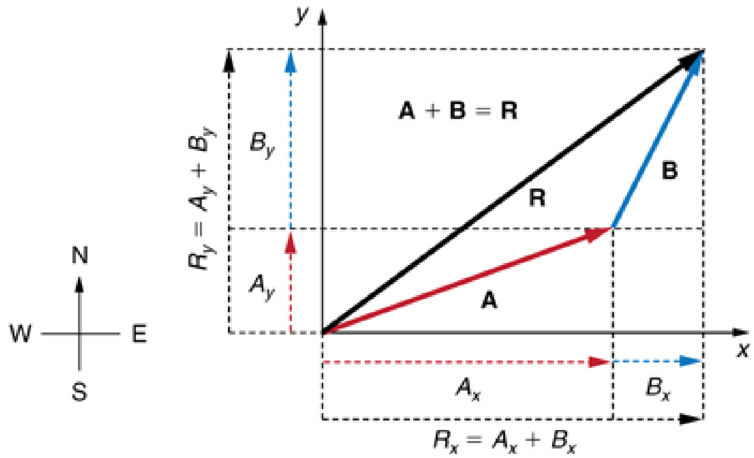
Equation:

$$R_x = A_x + B_x$$

and

Equation:

$$R_y = A_y + B_y.$$



The magnitude of the vectors $\mathbf{A}_x$ and $\mathbf{B}_x$ add to give the magnitude R_x of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors $\mathbf{A}_y$ and $\mathbf{B}_y$ add to give the magnitude R_y of the resultant vector in the vertical direction.

Components along the same axis, say the *x*-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the *y*-axis. (For example, a 9-block eastward walk could be taken in two legs, the first 3 blocks east and the second 6 blocks east, for a total of 9, because they are along the same direction.) So resolving vectors into components along common axes makes it easier to add them. Now that the components of $\mathbf{R}$ are known, its magnitude and direction can be found.

Step 3. *To get the magnitude R of the resultant, use the Pythagorean theorem:*

Equation:

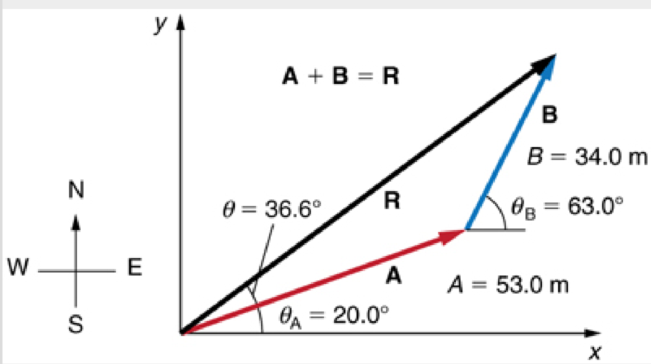
$$R = \sqrt{R_x^2 + R_y^2}.$$

Step 4. *To get the direction of the resultant:*
Equation:

$$\theta = \tan^{-1}(R_y/R_x).$$

The following example illustrates this technique for adding vectors using perpendicular components.

Example:
Adding Vectors Using Analytical Methods
Add the vector $\mathbf{A}$ to the vector $\mathbf{B}$ shown in [link], using perpendicular components along the *x*- and *y*-axes. The *x*- and *y*-axes are along the east–west and north–south directions, respectively. Vector $\mathbf{A}$ represents the first leg of a walk in which a person walks 53.0 m in a direction 20.0^o north of east. Vector $\mathbf{B}$ represents the second leg, a displacement of 34.0 m in a direction 63.0^o north of east.



Vector $\mathbf{A}$ has magnitude 53.0 m and direction 20.0^o north of the *x*-axis. Vector $\mathbf{B}$ has magnitude 34.0 m and direction 63.0^o north of the *x*-axis. You can use analytical methods to determine the magnitude and direction of $\mathbf{R}$.

Strategy

The components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*- and *y*-axes represent walking due east and due north to get to the same ending point. Once found, they are combined to produce the resultant.

Solution

Following the method outlined above, we first find the components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*- and *y*-axes. Note that $A = 53.0\text{ m}$, $\theta_{\text{A}} = 20.0^{\text{o}}$, $B = 34.0\text{ m}$, and $\theta_{\text{B}} = 63.0^{\text{o}}$. We find the *x*-components by using $A_x = A\cos\theta$, which gives

Equation:

$$\begin{aligned} A_x &= A\cos\theta_{\text{A}} = (53.0\text{ m})(\cos 20.0^{\text{o}}) \\ &= (53.0\text{ m})(0.940) = 49.8\text{ m} \end{aligned}$$

and

Equation:

$$\begin{aligned} B_x &= B\cos\theta_{\text{B}} = (34.0\text{ m})(\cos 63.0^{\text{o}}) \\ &= (34.0\text{ m})(0.454) = 15.4\text{ m}. \end{aligned}$$

Similarly, the *y*-components are found using $A_y = A\sin\theta_{\text{A}}$:

Equation:

$$\begin{aligned} A_y &= A\sin\theta_{\text{A}} = (53.0\text{ m})(\sin 20.0^{\text{o}}) \\ &= (53.0\text{ m})(0.342) = 18.1\text{ m} \end{aligned}$$

and

Equation:

$$\begin{aligned} B_y &= B\sin\theta_{\text{B}} = (34.0\text{ m})(\sin 63.0^{\text{o}}) \\ &= (34.0\text{ m})(0.891) = 30.3\text{ m}. \end{aligned}$$

The *x*- and *y*-components of the resultant are thus

Equation:

$$R_x = A_x + B_x = 49.8\text{ m} + 15.4\text{ m} = 65.2\text{ m}$$

and

Equation:

$$R_y = A_y + B_y = 18.1\text{ m} + 30.3\text{ m} = 48.4\text{ m}.$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem:

Equation:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2\text{ m}}$$

so that

Equation:

$$R = 81.2\text{ m}.$$

Finally, we find the direction of the resultant:

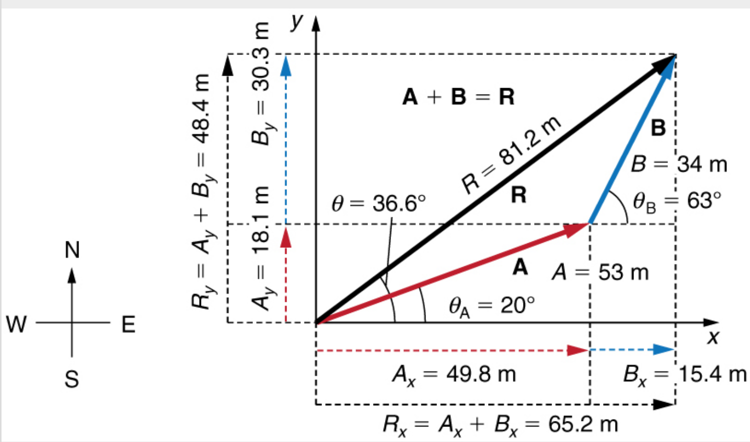
Equation:

$$\theta = \tan^{-1}(R_y/R_x) = +\tan^{-1}(48.4/65.2).$$

Thus,

Equation:

$$\theta = \tan^{-1}(0.742) = 36.6^{\circ}.$$



Using analytical methods, we see that the magnitude of $\mathbf{R}$ is 81.2 m and its

direction is 36.6° north of east.

Discussion
This example illustrates the addition of vectors using perpendicular components. Vector subtraction using perpendicular components is very similar—it is just the addition of a negative vector.
Subtraction of vectors is accomplished by the addition of a negative vector. That is, $\mathbf{A} - \mathbf{B} \equiv \mathbf{A} + (-\mathbf{B})$. Thus, *the method for the subtraction of vectors using perpendicular components is identical to that for addition*. The components of $-\mathbf{B}$ are the negatives of the components of $\mathbf{B}$. The *x*- and *y*-components of the resultant $\mathbf{A} - \mathrm{B} = \mathrm{R}$ are thus
Equation:

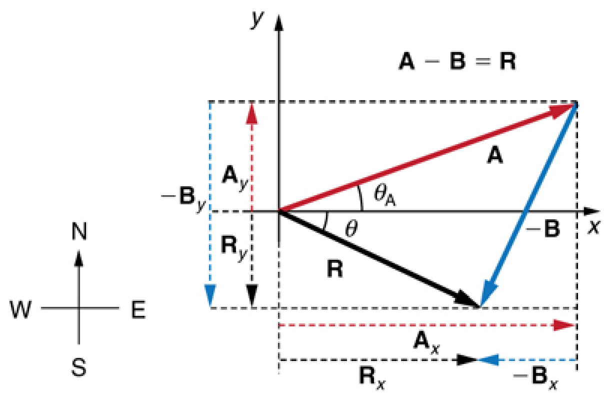
$$R_x = A_x + (-B_x)$$

and
Equation:

$$R_y = A_y + (-B_y)$$

and the rest of the method outlined above is identical to that for addition. (See [link].)

Analyzing vectors using perpendicular components is very useful in many areas of physics, because perpendicular quantities are often independent of one another. The next module, Projectile Motion, is one of many in which using perpendicular components helps make the picture clear and simplifies the physics.



The subtraction of the two vectors shown in [link]. The components of $-\mathbf{B}$ are the negatives of the components of $\mathbf{B}$. The method of subtraction is the same as that for addition.

Note:
PhET Explorations: Vector Addition
Learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.

Vector Addition

Summary

- The analytical method of vector addition and subtraction involves using the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.
- The steps to add vectors $\mathbf{A}$ and $\mathbf{B}$ using the analytical method are as follows:

  Step 1: Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations
  Equation:

$$\begin{aligned} A_x &= A \cos \theta \\ B_x &= B \cos \theta \end{aligned}$$

  and
  Equation:

$$\begin{aligned} A_y &= A \sin \theta \\ B_y &= B \sin \theta. \end{aligned}$$

  Step 2: Add the horizontal and vertical components of each vector to determine the components R_x and R_y of the resultant vector, $\mathbf{R}$:
  Equation:

$$R_x = A_x + B_x$$

  and
  Equation:

$$R_y = A_y + B_{y.}$$

  Step 3: Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector $\mathbf{R}$:
  Equation:

$$R = \sqrt{R_x^2 + R_y^2}.$$

Step 4: Use a trigonometric identity to determine the direction, θ, of $\mathbf{R}$
:

Equation:

$$\theta = \tan^{-1}(R_y / R_x).$$

Conceptual Questions

Exercise:

Problem:

Suppose you add two vectors $\mathbf{A}$ and $\mathbf{B}$. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude?

Exercise:

Problem:

Give an example of a nonzero vector that has a component of zero.

Exercise:

Problem:

Explain why a vector cannot have a component greater than its own magnitude.

Exercise:

Problem:

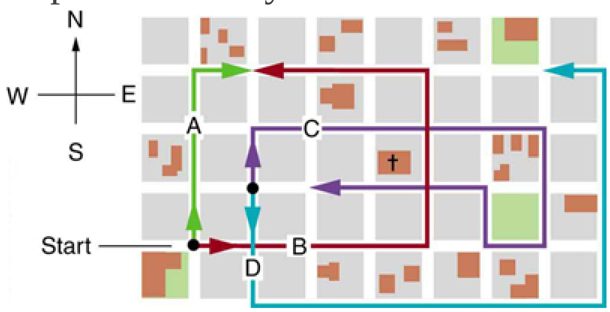
If the vectors $\mathbf{A}$ and $\mathbf{B}$ are perpendicular, what is the component of $\mathbf{A}$ along the direction of $\mathbf{B}$? What is the component of $\mathbf{B}$ along the direction of $\mathbf{A}$?

Problems & Exercises

Exercise:

Problem:

Find the following for path C in [link]: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.



The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

Solution:

(a) 1.56 km

(b) 120 m east

Exercise:

Problem:

Find the following for path D in [link]: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

Exercise:

Problem:

Find the north and east components of the displacement from San Francisco to Sacramento shown in [link].

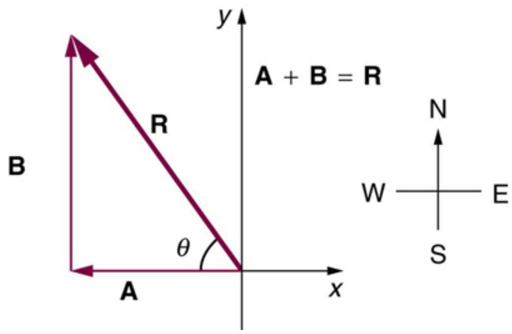
Solution:

North-component 87.0 km, east-component 87.0 km

Exercise:

Problem:

Solve the following problem using analytical techniques: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in [link], then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)



The two displacements $\mathbf{A}$ and $\mathbf{B}$ add to give a total displacement $\mathbf{R}$ having magnitude R and direction θ.

Note that you can also solve this graphically. Discuss why the analytical technique for solving this problem is potentially more accurate than the graphical technique.

Exercise:

Problem:

Repeat [link] using analytical techniques, but reverse the order of the two legs of the walk and show that you get the same final result. (This problem shows that adding them in reverse order gives the same result —that is, $\mathbf{B} + \mathbf{A} = \mathbf{A} + \mathbf{B}$.) Discuss how taking another path to reach the same point might help to overcome an obstacle blocking you other path.

Solution:

30.8 m, 35.8 west of north

Exercise:

Problem:

You drive $7.50\ \text{km}$ in a straight line in a direction 15° east of north. (a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (This determination is equivalent to find the components of the displacement along the east and north directions.) (b) Show that you still arrive at the same point if the east and north legs are reversed in order.

Exercise:

Problem:

Do [link] again using analytical techniques and change the second leg of the walk to $25.0\ \text{m}$ straight south. (This is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, finding $\mathbf{R}' = \mathbf{A} - \mathbf{B}$) (b) Repeat again, but now you first walk $25.0\ \text{m}$ north and then $18.0\ \text{m}$ east. (This is equivalent to subtract $\mathbf{A}$ from $\mathbf{B}$ —that is, to find $\mathbf{A} = \mathbf{B} + \mathbf{C}$. Is that consistent with your result?)

Solution:

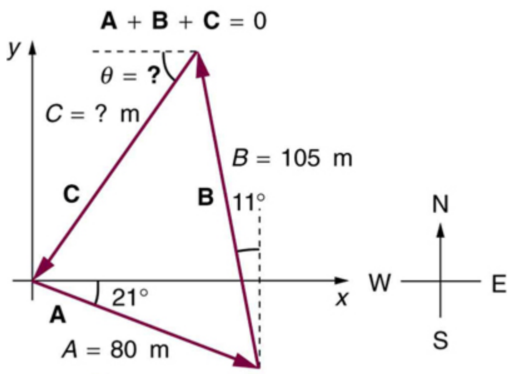
(a) $30.8\ \text{m}$, 54.2° south of west

(b) $30.8\ \text{m}$, 54.2° north of east

Exercise:

Problem:

A new landowner has a triangular piece of flat land she wishes to fence. Starting at the west corner, she measures the first side to be 80.0 m long and the next to be 105 m. These sides are represented as displacement vectors $\mathbf{A}$ from $\mathbf{B}$ in [link]. She then correctly calculates the length and orientation of the third side C. What is her result?



Exercise:

Problem:

You fly 32.0 km in a straight line in still air in the direction 35.0^{o} south of west. (a) Find the distances you would have to fly straight south and then straight west to arrive at the same point. (This determination is equivalent to finding the components of the displacement along the south and west directions.) (b) Find the distances you would have to fly first in a direction 45.0^{o} south of west and then in a direction 45.0^{o} west of north. These are the components of the displacement along a different set of axes—one rotated 45^{o}.

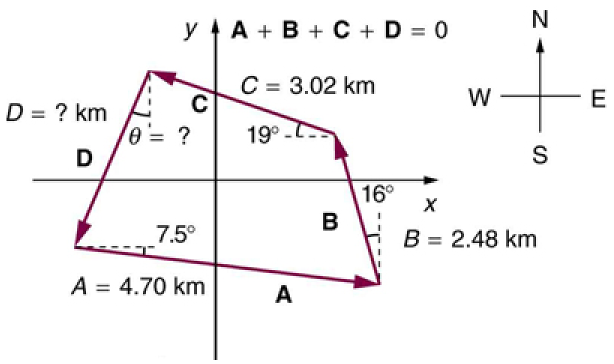
Solution:

18.4 km south, then 26.2 km west(b) 31.5 km at 45.0^{o} south of west, then 5.56 km at 45.0^{o} west of north

Exercise:

Problem:

A farmer wants to fence off his four-sided plot of flat land. He measures the first three sides, shown as $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ in [link], and then correctly calculates the length and orientation of the fourth side $\mathbf{D}$. What is his result?



Exercise:

Problem:

In an attempt to escape his island, Gilligan builds a raft and sets to sea. The wind shifts a great deal during the day, and he is blown along the following straight lines: $2.50\text{ km}\ 45.0^{\circ}$ north of west; then $4.70\text{ km}\ 60.0^{\circ}$ south of east; then $1.30\text{ km}\ 25.0^{\circ}$ south of west; then 5.10 km straight east; then $1.70\text{ km}\ 5.00^{\circ}$ east of north; then $7.20\text{ km}\ 55.0^{\circ}$ south of west; and finally $2.80\text{ km}\ 10.0^{\circ}$ north of east. What is his final position relative to the island?

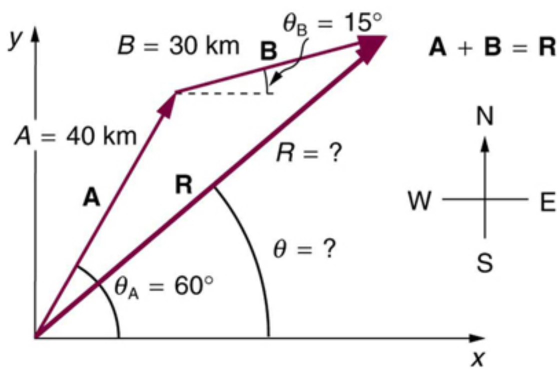
Solution:

7.34 km, 63.5° south of east

Exercise:

Problem:

Suppose a pilot flies 40.0 km in a direction 60° north of east and then flies 30.0 km in a direction 15° north of east as shown in [link]. Find her total distance R from the starting point and the direction θ of the straight-line path to the final position. Discuss qualitatively how this flight would be altered by a wind from the north and how the effect of the wind would depend on both wind speed and the speed of the plane relative to the air mass.



Glossary

analytical method
: the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

Addition of Velocities
Exercise:

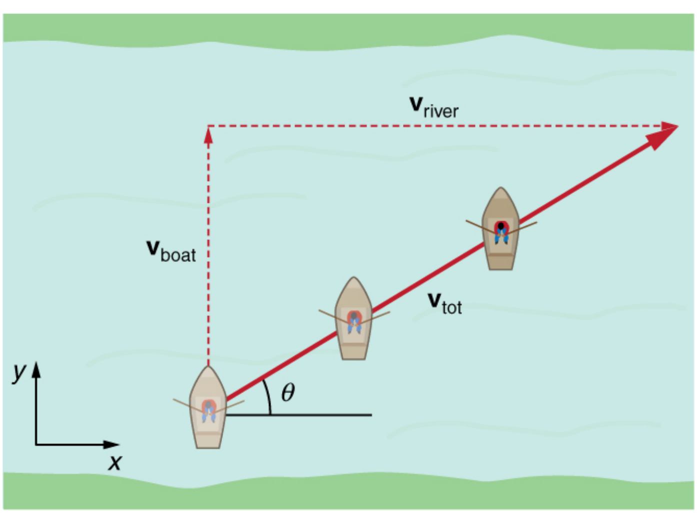
UMASS AMHERST Instructor's Notes

Problem:

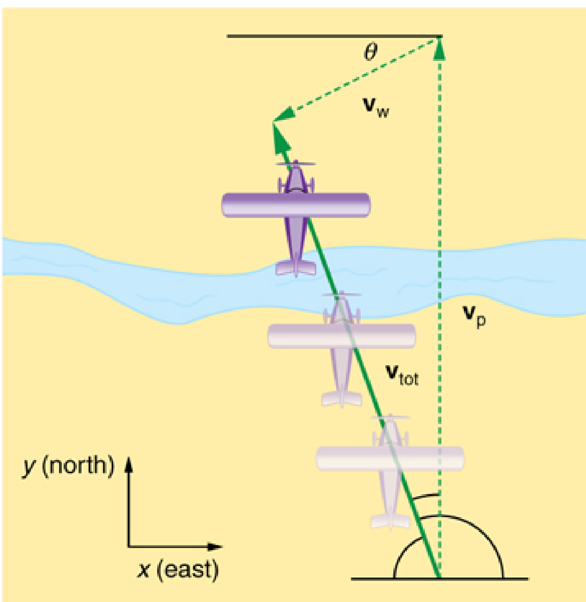
We will not being discussing this section in detail. This section is here primarily for your reference.

Relative Velocity

If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves *diagonally* relative to the shore, as in [link]. The boat does not move in the direction in which it is pointed. The reason, of course, is that the river carries the boat downstream. Similarly, if a small airplane flies overhead in a strong crosswind, you can sometimes see that the plane is not moving in the direction in which it is pointed, as illustrated in [link]. The plane is moving straight ahead relative to the air, but the movement of the air mass relative to the ground carries it sideways.



A boat trying to head straight across a river will actually move diagonally relative to the shore as shown. Its total velocity (solid arrow) relative to the shore is the sum of its velocity relative to the river plus the velocity of the river relative to the shore.



An airplane heading straight north is instead carried to the west and slowed down by wind. The plane does not move relative to the ground in the direction it points; rather, it moves in the direction of its total velocity (solid arrow).

In each of these situations, an object has a **velocity** relative to a medium (such as a river) and that medium has a velocity relative to an observer on solid ground. The velocity of the object *relative to the observer* is the sum of these velocity vectors, as indicated in [link] and [link]. These situations are only two of many in which it is useful to add velocities. In this module, we first re-examine how to add velocities and then consider certain aspects of what relative velocity means.

How do we add velocities? Velocity is a vector (it has both magnitude and direction); the rules of **vector addition** discussed in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods apply to the addition of velocities, just as they do for any other vectors. In one-dimensional motion, the addition of velocities is simple—they add like ordinary numbers. For example, if a field hockey player is moving at 5 m/s straight toward the goal and drives the ball in the same direction with a velocity of 30 m/s relative to her body, then the velocity of the ball is 35 m/s relative to the stationary, profusely sweating goalkeeper standing in front of the goal.

In two-dimensional motion, either graphical or analytical techniques can be used to add velocities. We will concentrate on analytical techniques. The following equations give the relationships between the magnitude and direction of velocity (v and θ) and its components (v_x and v_y) along the *x*- and *y*-axes of an appropriately chosen coordinate system:

Equation:

$$v_x = v \cos \theta$$

Equation:

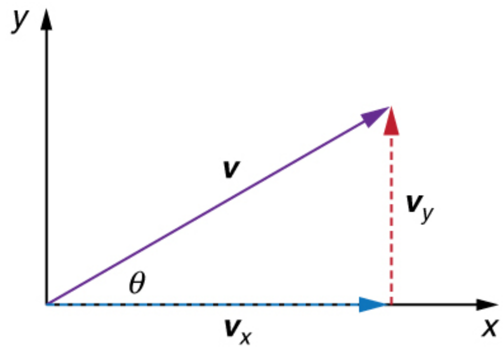
$$v_y = v \sin \theta$$

Equation:

$$v = \sqrt{v_x^2 + v_y^2}$$

Equation:

$$\theta = \tan^{-1}(v_y/v_x).$$



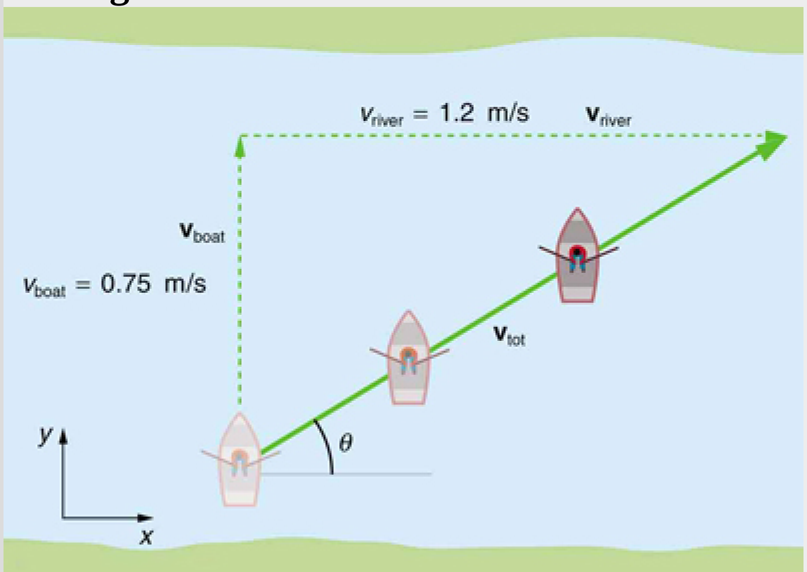
The velocity, v, of an object traveling at an angle θ to the horizontal axis is the sum of component vectors $\mathbf{v}_x$ and $\mathbf{v}_y$.

These equations are valid for any vectors and are adapted specifically for velocity. The first two equations are used to find the components of a velocity when its magnitude and direction are known. The last two are used to find the magnitude and direction of velocity when its components are known.

Note:
Take-Home Experiment: Relative Velocity of a Boat
Fill a bathtub half-full of water. Take a toy boat or some other object that floats in water. Unplug the drain so water starts to drain. Try pushing the boat from one side of the tub to the other and perpendicular to the flow of water. Which way do you need to push the boat so that it ends up

immediately opposite? Compare the directions of the flow of water, heading of the boat, and actual velocity of the boat.

Example:
Adding Velocities: A Boat on a River



A boat attempts to travel straight across a river at a speed 0.75 m/s. The current in the river, however, flows at a speed of 1.20 m/s to the right. What is the total displacement of the boat relative to the shore?

Refer to [link], which shows a boat trying to go straight across the river. Let us calculate the magnitude and direction of the boat's velocity relative to an observer on the shore, $\mathbf{v}_{\text{tot}}$. The velocity of the boat, $\mathbf{v}_{\text{boat}}$, is 0.75 m/s in the y-direction relative to the river and the velocity of the river, $\mathbf{v}_{\text{river}}$, is 1.20 m/s to the right.

Strategy

We start by choosing a coordinate system with its x-axis parallel to the velocity of the river, as shown in [link]. Because the boat is directed straight toward the other shore, its velocity relative to the water is parallel

to the y-axis and perpendicular to the velocity of the river. Thus, we can add the two velocities by using the equations $v_{\text{tot}} = \sqrt{v_x^2 + v_y^2}$ and $\theta = \tan^{-1}(v_y/v_x)$ directly.

Solution

The magnitude of the total velocity is

Equation:

$$v_{\text{tot}} = \sqrt{v_x^2 + v_y^2},$$

where

Equation:

$$v_x = v_{\text{river}} = 1.20\ \text{m/s}$$

and

Equation:

$$v_y = v_{\text{boat}} = 0.750\ \text{m/s}.$$

Thus,

Equation:

$$v_{\text{tot}} = \sqrt{(1.20\ \text{m/s})^2 + (0.750\ \text{m/s})^2}$$

yielding

Equation:

$$v_{\text{tot}} = 1.42\ \text{m/s}.$$

The direction of the total velocity θ is given by:

Equation:

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(0.750/1.20).$$

This equation gives

Equation:

$$\theta = 32.0^{\text{o}}.$$

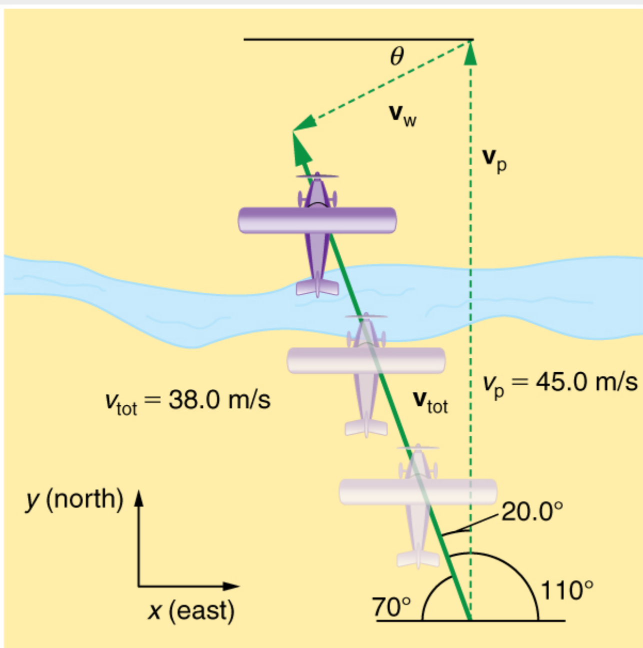
Discussion

Both the magnitude v and the direction θ of the total velocity are consistent with [link]. Note that because the velocity of the river is large compared with the velocity of the boat, it is swept rapidly downstream. This result is evidenced by the small angle (only 32.0^{o}) the total velocity has relative to the riverbank.

Example:

Calculating Velocity: Wind Velocity Causes an Airplane to Drift

Calculate the wind velocity for the situation shown in [link]. The plane is known to be moving at 45.0 m/s due north relative to the air mass, while its velocity relative to the ground (its total velocity) is 38.0 m/s in a direction 20.0^{o} west of north.



An airplane is known to be heading north at 45.0 m/s, though its velocity relative to the ground is 38.0 m/s at an angle west of north.

What is the speed and direction of the wind?

Strategy

In this problem, somewhat different from the previous example, we know the total velocity $\mathbf{v}_{\text{tot}}$ and that it is the sum of two other velocities, $\mathbf{v}_{\text{w}}$ (the wind) and $\mathbf{v}_{\text{p}}$ (the plane relative to the air mass). The quantity $\mathbf{v}_{\text{p}}$ is known, and we are asked to find $\mathbf{v}_{\text{w}}$. None of the velocities are perpendicular, but it is possible to find their components along a common set of perpendicular axes. If we can find the components of $\mathbf{v}_{\text{w}}$, then we can combine them to solve for its magnitude and direction. As shown in [link], we choose a coordinate system with its *x*-axis due east and its *y*-axis due north (parallel to $\mathbf{v}_{\text{p}}$). (You may wish to look back at the discussion of the addition of vectors using perpendicular components in Vector Addition and Subtraction: Analytical Methods.)

Solution

Because $\mathbf{v}_{\text{tot}}$ is the vector sum of the $\mathbf{v}_{\text{w}}$ and $\mathbf{v}_{\text{p}}$, its *x*- and *y*-components are the sums of the *x*- and *y*-components of the wind and plane velocities. Note that the plane only has vertical component of velocity so $v_{\text{p}x} = 0$ and $v_{\text{p}y} = v_{\text{p}}$. That is,

Equation:

$$v_{\text{tot}x} = v_{\text{w}x}$$

and

Equation:

$$v_{\text{tot}y} = v_{\text{w}y} + v_{\text{p}}.$$

We can use the first of these two equations to find $v_{\text{w}x}$:

Equation:

$$v_{\text{w}y} = v_{\text{tot}x} = v_{\text{tot}}\text{cos } 110^{\text{o}}.$$

Because $v_{\text{tot}} = 38.0 \text{ m/s}$ and $\text{cos } 110^{\text{o}} =\text{–} 0.342$ we have

Equation:

$$v_{\text{w}y} = (38.0 \text{ m/s})(\text{–}0.342) = \text{–}13 \text{ m/s}.$$

The minus sign indicates motion west which is consistent with the diagram.
Now, to find v_{wy} we note that
Equation:

$$v_{\mathrm{tot}y} = v_{\mathrm{w}y} + v_{\mathrm{p}}$$

Here $v_{\mathrm{tot}y} = v_{\mathrm{tot}}$sin 110º; thus,
Equation:

$$v_{\mathrm{w}y} = (38.0\ \mathrm{m/s})(0.940) - 45.0\ \mathrm{m/s} = -9.29\ \mathrm{m/s}.$$

This minus sign indicates motion south which is consistent with the diagram.
Now that the perpendicular components of the wind velocity $v_{\mathrm{w}x}$ and $v_{\mathrm{w}y}$ are known, we can find the magnitude and direction of $\mathbf{v}_{\mathrm{w}}$. First, the magnitude is
Equation:

$$\begin{array}{rcl} v_{\mathrm{w}} & = & \sqrt{v_{\mathrm{w}x}^2 + v_{\mathrm{w}y}^2} \\ & = & \sqrt{(-13.0\ \mathrm{m/s})^2 + (-9.29\ \mathrm{m/s})^2} \end{array}$$

so that
Equation:

$$v_{\mathrm{w}} = 16.0\ \mathrm{m/s}.$$

The direction is:
Equation:

$$\theta = \tan^{-1}(v_{\mathrm{w}y}/v_{\mathrm{w}x}) = \tan^{-1}(-9.29/-13.0)$$

giving
Equation:

$$\theta = 35.6^{\mathrm{o}}.$$

Discussion
The wind’s speed and direction are consistent with the significant effect the wind has on the total velocity of the plane, as seen in [link]. Because the plane is fighting a strong combination of crosswind and head-wind, it ends up with a total velocity significantly less than its velocity relative to the air mass as well as heading in a different direction.

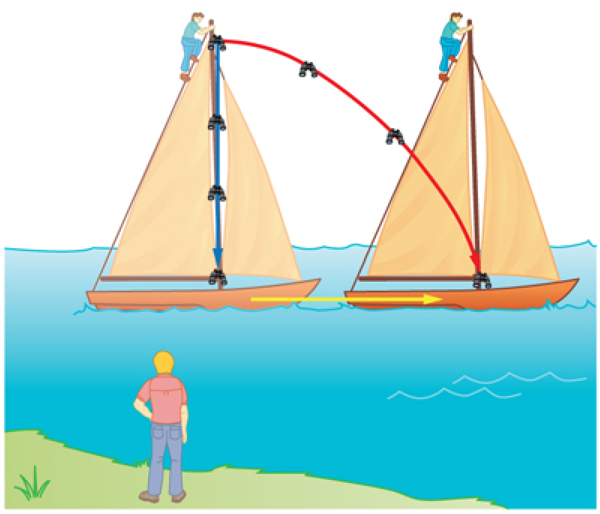
Note that in both of the last two examples, we were able to make the mathematics easier by choosing a coordinate system with one axis parallel to one of the velocities. We will repeatedly find that choosing an appropriate coordinate system makes problem solving easier. For example, in projectile motion we always use a coordinate system with one axis parallel to gravity.

Relative Velocities and Classical Relativity

When adding velocities, we have been careful to specify that the *velocity is relative to some reference frame*. These velocities are called **relative velocities**. For example, the velocity of an airplane relative to an air mass is different from its velocity relative to the ground. Both are quite different from the velocity of an airplane relative to its passengers (which should be close to zero). Relative velocities are one aspect of **relativity**, which is defined to be the study of how different observers moving relative to each other measure the same phenomenon.

Nearly everyone has heard of relativity and immediately associates it with Albert Einstein (1879–1955), the greatest physicist of the 20th century. Einstein revolutionized our view of nature with his *modern* theory of relativity, which we shall study in later chapters. The relative velocities in this section are actually aspects of classical relativity, first discussed correctly by Galileo and Isaac Newton. **Classical relativity** is limited to situations where speeds are less than about 1% of the speed of light—that is, less than $3,000\ \text{km/s}$. Most things we encounter in daily life move slower than this speed.

Let us consider an example of what two different observers see in a situation analyzed long ago by Galileo. Suppose a sailor at the top of a mast on a moving ship drops his binoculars. Where will it hit the deck? Will it hit at the base of the mast, or will it hit behind the mast because the ship is moving forward? The answer is that if air resistance is negligible, the binoculars will hit at the base of the mast at a point directly below its point of release. Now let us consider what two different observers see when the binoculars drop. One observer is on the ship and the other on shore. The binoculars have no horizontal velocity relative to the observer on the ship, and so he sees them fall straight down the mast. (See [link].) To the observer on shore, the binoculars and the ship have the *same* horizontal velocity, so both move the same distance forward while the binoculars are falling. This observer sees the curved path shown in [link]. Although the paths look different to the different observers, each sees the same result—the binoculars hit at the base of the mast and not behind it. To get the correct description, it is crucial to correctly specify the velocities relative to the observer.

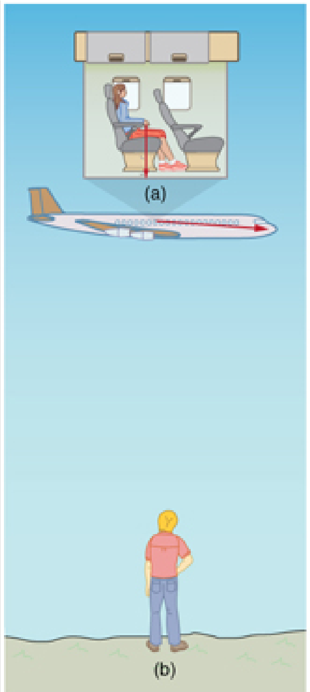


Classical relativity. The same motion as viewed by two different observers. An observer on the moving ship sees the binoculars dropped from the top

of its mast fall straight down. An observer on shore sees the binoculars take the curved path, moving forward with the ship. Both observers see the binoculars strike the deck at the base of the mast. The initial horizontal velocity is different relative to the two observers. (The ship is shown moving rather fast to emphasize the effect.)

Example:
Calculating Relative Velocity: An Airline Passenger Drops a Coin

An airline passenger drops a coin while the plane is moving at 260 m/s. What is the velocity of the coin when it strikes the floor 1.50 m below its point of release: (a) Measured relative to the plane? (b) Measured relative to the Earth?



The motion of a coin dropped inside an airplane as viewed by two different observers. (a) An observer in the plane sees the coin fall straight down. (b) An observer on the ground sees the coin move almost horizontally.

Strategy

Both problems can be solved with the techniques for falling objects and projectiles. In part (a), the initial velocity of the coin is zero relative to the plane, so the motion is that of a falling object (one-dimensional). In part (b), the initial velocity is 260 m/s horizontal relative to the Earth and gravity is vertical, so this motion is a projectile motion. In both parts, it is best to use a coordinate system with vertical and horizontal axes.

Solution for (a)

Using the given information, we note that the initial velocity and position are zero, and the final position is 1.50 m. The final velocity can be found using the equation:

Equation:

$$v_y{}^2 = v_{0y}{}^2 - 2g(y - y_0).$$

Substituting known values into the equation, we get

Equation:

$$v_y{}^2 = 0^2 - 2(9.80\ \mathrm{m/s^2})(-1.50\ \mathrm{m} - 0\ \mathrm{m}) = 29.4\ \mathrm{m^2/s^2}$$

yielding

Equation:

$$v_y = -5.42\ \mathrm{m/s}.$$

We know that the square root of 29.4 has two roots: 5.42 and -5.42. We choose the negative root because we know that the velocity is directed downwards, and we have defined the positive direction to be upwards. There is no initial horizontal velocity relative to the plane and no horizontal acceleration, and so the motion is straight down relative to the plane.

Solution for (b)

Because the initial vertical velocity is zero relative to the ground and vertical motion is independent of horizontal motion, the final vertical velocity for the coin relative to the ground is $v_y = -5.42\ \mathrm{m/s}$, the same as found in part (a). In contrast to part (a), there now is a horizontal component of the velocity. However, since there is no horizontal acceleration, the initial and final horizontal velocities are the same and

$v_x = 260\text{ m/s}$. The *x*- and *y*-components of velocity can be combined to find the magnitude of the final velocity:
Equation:

$$v = \sqrt{{v_x}^2 + {v_y}^2}.$$

Thus,
Equation:

$$v = \sqrt{(260\text{ m/s})^2 + (-5.42\text{ m/s})^2}$$

yielding
Equation:

$$v = 260.06\text{ m/s}.$$

The direction is given by:
Equation:

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(-5.42/260)$$

so that
Equation:

$$\theta = \tan^{-1}(-0.0208) = -1.19^{\text{o}}.$$

Discussion
In part (a), the final velocity relative to the plane is the same as it would be if the coin were dropped from rest on the Earth and fell 1.50 m. This result fits our experience; objects in a plane fall the same way when the plane is flying horizontally as when it is at rest on the ground. This result is also true in moving cars. In part (b), an observer on the ground sees a much different motion for the coin. The plane is moving so fast horizontally to begin with that its final velocity is barely greater than the initial velocity. Once again, we see that in two dimensions, vectors do not add like ordinary numbers—the final velocity v in part (b) is *not* $(260 – 5.42)\text{ m/s}$; rather, it is 260.06 m/s. The velocity's magnitude had to be calculated to

five digits to see any difference from that of the airplane. The motions as seen by different observers (one in the plane and one on the ground) in this example are analogous to those discussed for the binoculars dropped from the mast of a moving ship, except that the velocity of the plane is much larger, so that the two observers see *very* different paths. (See [link].) In addition, both observers see the coin fall 1.50 m vertically, but the one on the ground also sees it move forward 144 m (this calculation is left for the reader). Thus, one observer sees a vertical path, the other a nearly horizontal path.

Note:

Making Connections: Relativity and Einstein

Because Einstein was able to clearly define how measurements are made (some involve light) and because the speed of light is the same for all observers, the outcomes are spectacularly unexpected. Time varies with observer, energy is stored as increased mass, and more surprises await.

Note:

PhET Explorations: Motion in 2D

Try the new "Ladybug Motion 2D" simulation for the latest updated version. Learn about position, velocity, and acceleration vectors. Move the ball with the mouse or let the simulation move the ball in four types of motion (2 types of linear, simple harmonic, circle).

Motion in 2D

Summary

- Velocities in two dimensions are added using the same analytical vector techniques, which are rewritten as
  Equation:
  $$v_x = v \cos \theta$$
  Equation:
  $$v_y = v \sin \theta$$
  Equation:
  $$v = \sqrt{v_x^2 + v_y^2}$$
  Equation:
  $$\theta = \tan^{-1}(v_y/v_x).$$
- **Relative velocity** is the velocity of an object as observed from a particular reference frame, and it varies dramatically with reference frame.
- **Relativity** is the study of how different observers measure the same phenomenon, particularly when the observers move relative to one another. **Classical relativity** is limited to situations where speed is less than about 1% of the speed of light (3000 km/s).

Conceptual Questions

Exercise:

Problem:

What frame or frames of reference do you instinctively use when driving a car? When flying in a commercial jet airplane?

Exercise:

Problem:

A basketball player dribbling down the court usually keeps his eyes fixed on the players around him. He is moving fast. Why doesn't he need to keep his eyes on the ball?

Exercise:

Problem:

If someone is riding in the back of a pickup truck and throws a softball straight backward, is it possible for the ball to fall straight down as viewed by a person standing at the side of the road? Under what condition would this occur? How would the motion of the ball appear to the person who threw it?

Exercise:

Problem:

The hat of a jogger running at constant velocity falls off the back of his head. Draw a sketch showing the path of the hat in the jogger's frame of reference. Draw its path as viewed by a stationary observer.

Exercise:

Problem:

A clod of dirt falls from the bed of a moving truck. It strikes the ground directly below the end of the truck. What is the direction of its velocity relative to the truck just before it hits? Is this the same as the direction of its velocity relative to ground just before it hits? Explain your answers.

Problems & Exercises

Exercise:

Problem:

Bryan Allen pedaled a human-powered aircraft across the English Channel from the cliffs of Dover to Cap Gris-Nez on June 12, 1979. (a) He flew for 169 min at an average velocity of 3.53 m/s in a direction 45º south of east. What was his total displacement? (b) Allen encountered a headwind averaging 2.00 m/s almost precisely in the opposite direction of his motion relative to the Earth. What was his average velocity relative to the air? (c) What was his total displacement relative to the air mass?

Solution:

(a) 35.8 km, 45º south of east

(b) 5.53 m/s, 45º south of east

(c) 56.1 km, 45º south of east

Exercise:

Problem:

A seagull flies at a velocity of 9.00 m/s straight into the wind. (a) If it takes the bird 20.0 min to travel 6.00 km relative to the Earth, what is the velocity of the wind? (b) If the bird turns around and flies with the wind, how long will he take to return 6.00 km? (c) Discuss how the wind affects the total round-trip time compared to what it would be with no wind.

Exercise:

Problem:

Near the end of a marathon race, the first two runners are separated by a distance of 45.0 m. The front runner has a velocity of 3.50 m/s, and the second a velocity of 4.20 m/s. (a) What is the velocity of the second runner relative to the first? (b) If the front runner is 250 m from the finish line, who will win the race, assuming they run at constant velocity? (c) What distance ahead will the winner be when she crosses the finish line?

Solution:

(a) 0.70 m/s faster

(b) Second runner wins

(c) 4.17 m

Exercise:

Problem:

Verify that the coin dropped by the airline passenger in the [link] travels 144 m horizontally while falling 1.50 m in the frame of reference of the Earth.

Exercise:

Problem:

A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. The ball is thrown at an angle of 25.0° relative to the ground and is caught at the same height as it is released. What is the initial velocity of the ball *relative to the quarterback* ?

Solution:

$17.0 \text{ m/s}, 22.1^{\circ}$

Exercise:

Problem:

A ship sets sail from Rotterdam, The Netherlands, heading due north at 7.00 m/s relative to the water. The local ocean current is 1.50 m/s in a direction 40.0° north of east. What is the velocity of the ship relative to the Earth?

Exercise:

Problem:

(a) A jet airplane flying from Darwin, Australia, has an air speed of 260 m/s in a direction 5.0° south of west. It is in the jet stream, which is blowing at 35.0 m/s in a direction 15° south of east. What is the velocity of the airplane relative to the Earth? (b) Discuss whether your answers are consistent with your expectations for the effect of the wind on the plane's path.

Solution:

(a) 230 m/s, 8.0° south of west

(b) The wind should make the plane travel slower and more to the south, which is what was calculated.

Exercise:

Problem:

(a) In what direction would the ship in [link] have to travel in order to have a velocity straight north relative to the Earth, assuming its speed relative to the water remains 7.00 m/s? (b) What would its speed be relative to the Earth?

Exercise:

Problem:

(a) Another airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction 20° south of east (as in [link]). Its direction of motion relative to the Earth is 45.0° south of west, while its direction of travel relative to the air is 5.00° south of west. What is the airplane's speed relative to the air mass? (b) What is the airplane's speed relative to the Earth?

Solution:

(a) 63.5 m/s

(b) 29.6 m/s

Exercise:

Problem:

A sandal is dropped from the top of a 15.0-m-high mast on a ship moving at 1.75 m/s due south. Calculate the velocity of the sandal when it hits the deck of the ship: (a) relative to the ship and (b) relative to a stationary observer on shore. (c) Discuss how the answers give a consistent result for the position at which the sandal hits the deck.

Exercise:

Problem:

The velocity of the wind relative to the water is crucial to sailboats. Suppose a sailboat is in an ocean current that has a velocity of 2.20 m/s in a direction 30.0° east of north relative to the Earth. It encounters a wind that has a velocity of 4.50 m/s in a direction of 50.0° south of west relative to the Earth. What is the velocity of the wind relative to the water?

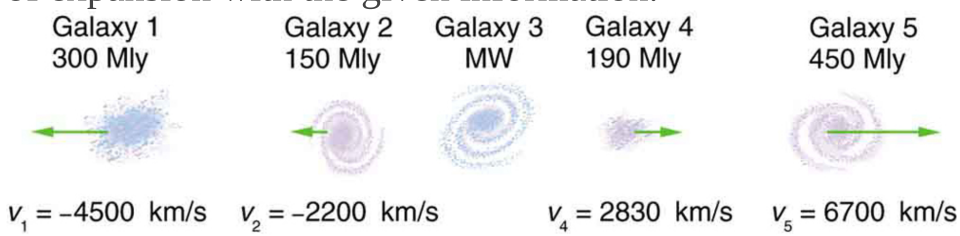
Solution:

$6.68 \text{ m/s}, 53.3^{\circ}$ south of west

Exercise:

Problem:

The great astronomer Edwin Hubble discovered that all distant galaxies are receding from our Milky Way Galaxy with velocities proportional to their distances. It appears to an observer on the Earth that we are at the center of an expanding universe. [link] illustrates this for five galaxies lying along a straight line, with the Milky Way Galaxy at the center. Using the data from the figure, calculate the velocities: (a) relative to galaxy 2 and (b) relative to galaxy 5. The results mean that observers on all galaxies will see themselves at the center of the expanding universe, and they would likely be aware of relative velocities, concluding that it is not possible to locate the center of expansion with the given information.



Five galaxies on a straight line, showing their distances and velocities relative to the Milky Way (MW) Galaxy. The distances are in millions of light years (Mly), where a light year is the distance light travels in one year. The velocities are nearly proportional to the distances. The sizes of the galaxies are greatly exaggerated; an average galaxy is about 0.1 Mly across.

Exercise:

Problem:

(a) Use the distance and velocity data in [link] to find the rate of expansion as a function of distance.

(b) If you extrapolate back in time, how long ago would all of the galaxies have been at approximately the same position? The two parts of this problem give you some idea of how the Hubble constant for universal expansion and the time back to the Big Bang are determined, respectively.

Solution:

(a) $H_{\text{average}} = 14.9\frac{\text{km/s}}{\text{Mly}}$

(b) 20.2 billion years

Exercise:

Problem:

An athlete crosses a 25-m-wide river by swimming perpendicular to the water current at a speed of 0.5 m/s relative to the water. He reaches the opposite side at a distance 40 m downstream from his starting point. How fast is the water in the river flowing with respect to the ground? What is the speed of the swimmer with respect to a friend at rest on the ground?

Exercise:

Problem:

A ship sailing in the Gulf Stream is heading 25.0° west of north at a speed of 4.00 m/s relative to the water. Its velocity relative to the Earth is 4.80 m/s 5.00° west of north. What is the velocity of the Gulf Stream? (The velocity obtained is typical for the Gulf Stream a few hundred kilometers off the east coast of the United States.)

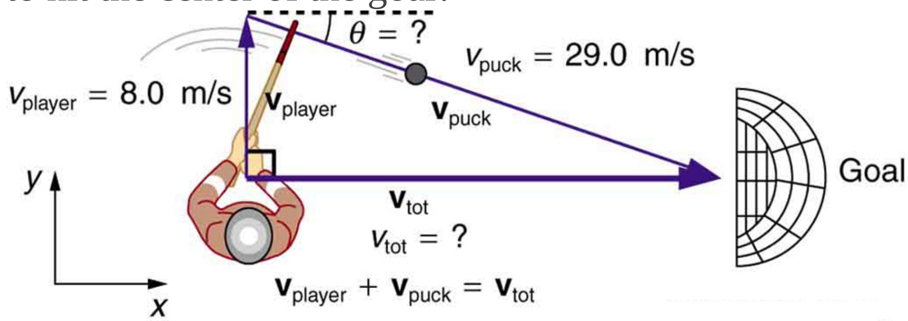
Solution:

1.72 m/s, 42.3° north of east

Exercise:

Problem:

An ice hockey player is moving at 8.00 m/s when he hits the puck toward the goal. The speed of the puck relative to the player is 29.0 m/s. The line between the center of the goal and the player makes a 90.0º angle relative to his path as shown in [link]. What angle must the puck's velocity make relative to the player (in his frame of reference) to hit the center of the goal?



An ice hockey player moving across the rink must shoot backward to give the puck a velocity toward the goal.

Exercise:

Problem:

Unreasonable Results Suppose you wish to shoot supplies straight up to astronauts in an orbit 36,000 km above the surface of the Earth. (a) At what velocity must the supplies be launched? (b) What is unreasonable about this velocity? (c) Is there a problem with the relative velocity between the supplies and the astronauts when the supplies reach their maximum height? (d) Is the premise unreasonable or is the available equation inapplicable? Explain your answer.

Exercise:

Problem:

Unreasonable Results A commercial airplane has an air speed of 280 m/s due east and flies with a strong tailwind. It travels 3000 km in a direction 5° south of east in 1.50 h. (a) What was the velocity of the plane relative to the ground? (b) Calculate the magnitude and direction of the tailwind's velocity. (c) What is unreasonable about both of these velocities? (d) Which premise is unreasonable?

Exercise:

Problem:

Construct Your Own Problem Consider an airplane headed for a runway in a cross wind. Construct a problem in which you calculate the angle the airplane must fly relative to the air mass in order to have a velocity parallel to the runway. Among the things to consider are the direction of the runway, the wind speed and direction (its velocity) and the speed of the plane relative to the air mass. Also calculate the speed of the airplane relative to the ground. Discuss any last minute maneuvers the pilot might have to perform in order for the plane to land with its wheels pointing straight down the runway.

Glossary

classical relativity
: the study of relative velocities in situations where speeds are less than about 1% of the speed of light—that is, less than 3000 km/s

relative velocity
: the velocity of an object as observed from a particular reference frame

relativity
: the study of how different observers moving relative to each other measure the same phenomenon

velocity
: speed in a given direction

vector addition

the rules that apply to adding vectors together

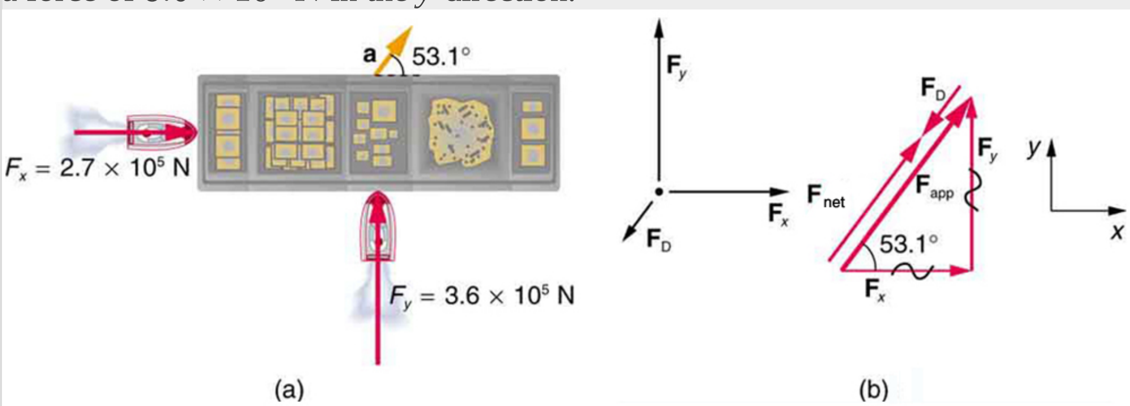
Introduction

This chapter covers some problem-solving strategies and a few examples of problems when working with forces. We will be going over these in class, so this section is entirely for your reference. If you do feel like you need more practice with force problems and free body diagrams, or if you are looking for a way to study for the exam, these sections are a good place to start. However, if you do so, it is highly recommended that you work through the problems yourselves as well. While reading about it alone can be somewhat helpful, you will get a lot more out of it if you work through the physics alongside.

Problem Solving Strategy

There are many interesting applications of Newton’s laws of motion, a few more of which are presented in this section. These serve also to illustrate some further subtleties of physics and to help build problem-solving skills.

Example:
Drag Force on a Barge
Suppose two tugboats push on a barge at different angles, as shown in [link]. The first tugboat exerts a force of 2.7×10^5 N in the *x*-direction, and the second tugboat exerts a force of 3.6×10^5 N in the *y*-direction.



(a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the *x*- and *y*-axes are in the same direction as $\mathbf{F}_x$ and $\mathbf{F}_y$. The problem quickly becomes a one-dimensional problem along the direction of $\mathbf{F}_{\text{app}}$, since friction is in the direction opposite to $\mathbf{F}_{\text{app}}$.

If the mass of the barge is 5.0×10^6 kg and its acceleration is observed to be $7.5 \times 10^{-2}\ \text{m/s}^2$ in the direction shown, what is the drag force of the water on the barge resisting the motion? (Note: drag force is a frictional force exerted by fluids, such as air or water. The drag force opposes the motion of the object.)
Strategy
The directions and magnitudes of acceleration and the applied forces are given in [link]**(a)**. We will define the total force of the tugboats on the barge as $\mathbf{F}_{\text{app}}$ so that:
Equation:

$$\mathbf{F}_{\text{app}}=\mathbf{F}_x+\mathbf{F}_y$$

Since the barge is flat bottomed, the drag of the water $\mathbf{F}_{\text{D}}$ will be in the direction opposite to $\mathbf{F}_{\text{app}}$, as shown in the free-body diagram in [link](b). The system of interest here is the barge, since the forces on *it* are given as well as its acceleration. Our strategy is to find the magnitude and direction of the net applied force $\mathbf{F}_{\text{app}}$, and then apply Newton's second law to solve for the drag force $\mathbf{F}_{\text{D}}$.

Solution

Since $\mathbf{F}_x$ and $\mathbf{F}_y$ are perpendicular, the magnitude and direction of $\mathbf{F}_{\text{app}}$ are easily found. First, the resultant magnitude is given by the Pythagorean theorem:

Equation:

$$\begin{aligned} F_{\text{app}} &= \sqrt{\mathrm{F}_x^2+\mathrm{F}_y^2} \\ F_{\text{app}} &= \sqrt{(2.7\times10^5\text{ N})^2+(3.6\times10^5\text{ N})^2} \;=\; 4.5\times10^5\text{ N}. \end{aligned}$$

The angle is given by

Equation:

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{F_y}{F_x}\right) \\ \theta &= \tan^{-1}\left(\frac{3.6\times10^5\text{ N}}{2.7\times10^5\text{ N}}\right)=53^{\text{o}}, \end{aligned}$$

which we know, because of Newton's first law, is the same direction as the acceleration. $\mathbf{F}_{\text{D}}$ is in the opposite direction of $\mathbf{F}_{\text{app}}$, since it acts to slow down the acceleration. Therefore, the net external force is in the same direction as $\mathbf{F}_{\text{app}}$, but its magnitude is slightly less than $\mathbf{F}_{\text{app}}$. The problem is now one-dimensional. From [link]**(b)**, we can see that

Equation:

$$F_{\text{net}}=F_{\text{app}}-F_{\text{D}}.$$

But Newton's second law states that

Equation:

$$F_{\text{net}}=\text{ma}.$$

Thus,

Equation:

$$F_{\text{app}}-F_{\text{D}}=\text{ma}.$$

This can be solved for the magnitude of the drag force of the water F_{D} in terms of known quantities:

Equation:

$$F_{\text{D}} = F_{\text{app}} - \text{ma}.$$

Substituting known values gives

Equation:

$$\text{F}_{\text{D}} = (4.5 \times 10^{5}\ \text{N}) - (5.0 \times 10^{6}\ \text{kg})(7.5 \times 10^{-2}\ \text{m/s}^{2}) = 7.5 \times 10^{4}\ \text{N}.$$

The direction of $\mathbf{F}_{\text{D}}$ has already been determined to be in the direction opposite to $\mathbf{F}_{\text{app}}$, or at an angle of 53^{o} south of west.

Discussion

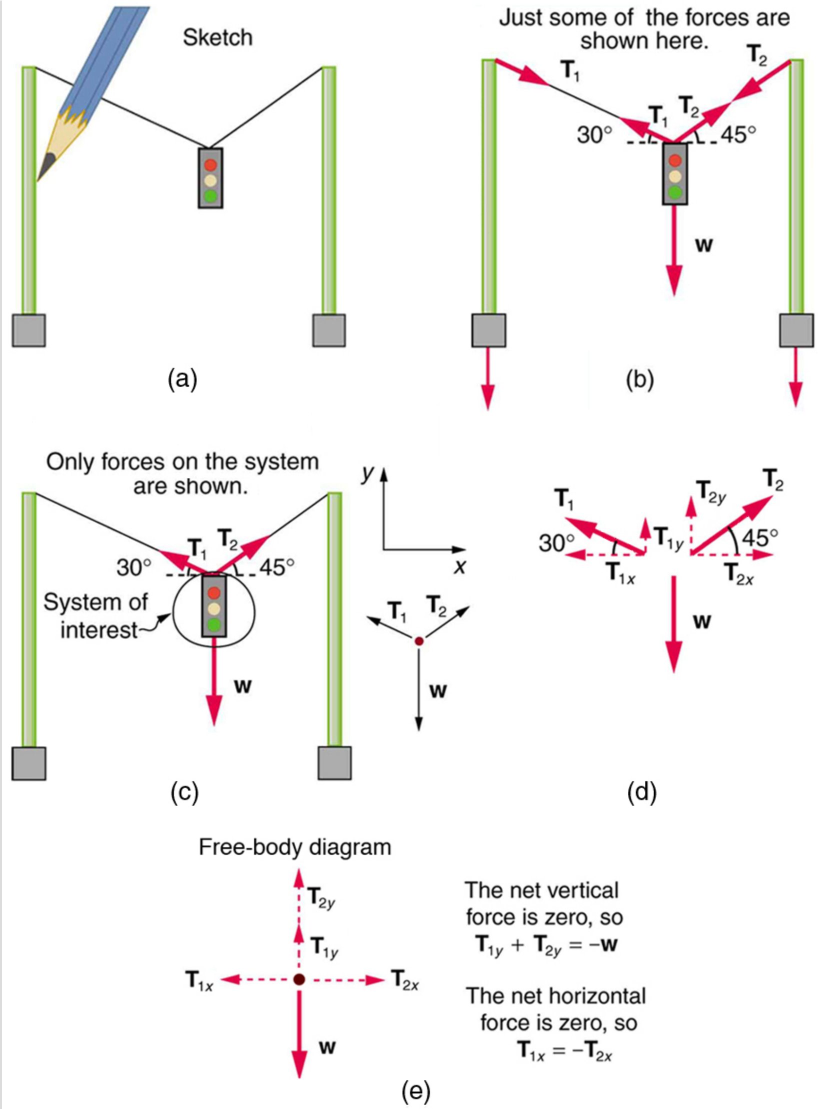
The numbers used in this example are reasonable for a moderately large barge. It is certainly difficult to obtain larger accelerations with tugboats, and small speeds are desirable to avoid running the barge into the docks. Drag is relatively small for a well-designed hull at low speeds, consistent with the answer to this example, where F_{D} is less than 1/600th of the weight of the ship.

In the earlier example of a tightrope walker we noted that the tensions in wires supporting a mass were equal only because the angles on either side were equal. Consider the following example, where the angles are not equal; slightly more trigonometry is involved.

Example:

Different Tensions at Different Angles

Consider the traffic light (mass 15.0 kg) suspended from two wires as shown in [link]. Find the tension in each wire, neglecting the masses of the wires.



A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical (*y*) and horizontal (*x*) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e) The free-body diagram shows the vertical and horizontal forces acting on the traffic light.

Strategy

The system of interest is the traffic light, and its free-body diagram is shown in [link] (c). The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in part (d) of the figure. There are two unknowns in this problem (T_1 and T_2), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

Solution

First consider the horizontal or *x*-axis:

Equation:

$$F_{\text{net}x} = T_{2x} - T_{1x} = 0.$$

Thus, as you might expect,

Equation:

$$T_{1x} = T_{2x}.$$

This gives us the following relationship between T_1 and T_2:

Equation:

$$T_1 \cos\left(30^\circ\right) = T_2 \cos\left(45^\circ\right).$$

Thus,

Equation:

$$T_2 = (1.225)T_1.$$

Note that T_1 and T_2 are not equal in this case, because the angles on either side are not equal. It is reasonable that T_2 ends up being greater than T_1, because it is exerted more vertically than T_1.

Now consider the force components along the vertical or *y*-axis:

Equation:

$$F_{\text{net }y} = T_{1y} + T_{2y} - w = 0.$$

This implies

Equation:

$$T_{1y} + T_{2y} = w.$$

Substituting the expressions for the vertical components gives

Equation:

$$T_1 \sin(30^\circ) + T_2 \sin(45^\circ) = w.$$

There are two unknowns in this equation, but substituting the expression for T_2 in terms of T_1 reduces this to one equation with one unknown:
Equation:

$$T_1(0.500) + (1.225T_1)(0.707) = w = \text{mg},$$

which yields
Equation:

$$(1.366)T_1 = (15.0 \text{ kg})(9.80 \text{ m/s}^2).$$

Solving this last equation gives the magnitude of T_1 to be
Equation:

$$T_1 = 108 \text{ N}.$$

Finally, the magnitude of T_2 is determined using the relationship between them, T_2 = 1.225 T_1, found above. Thus we obtain
Equation:

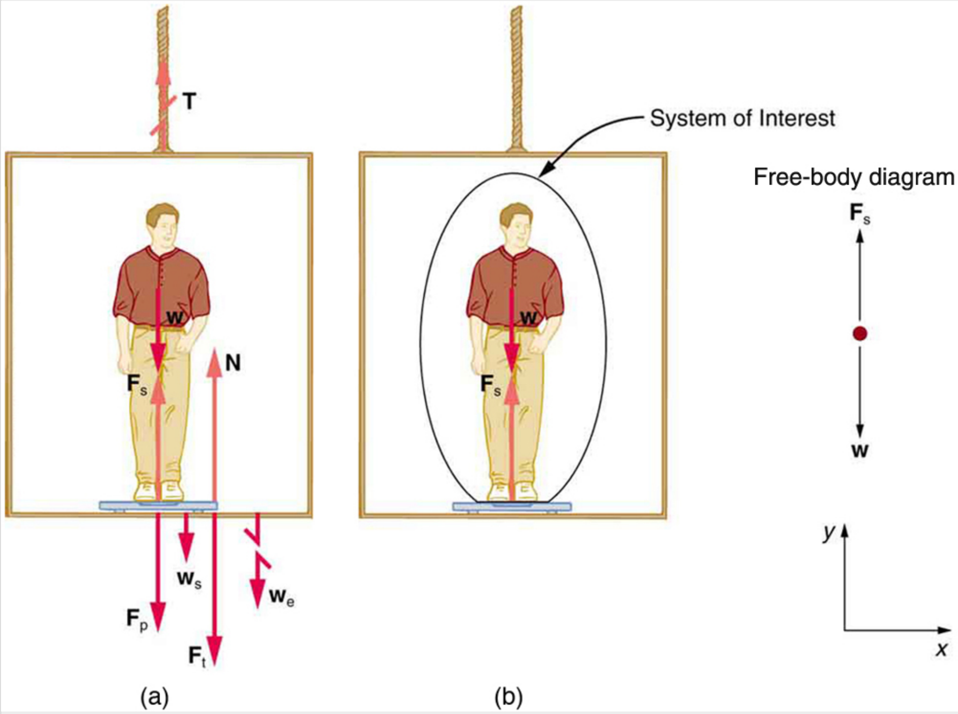
$$T_2 = 132 \text{ N}.$$

Discussion
Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker).

The bathroom scale is an excellent example of a normal force acting on a body. It provides a quantitative reading of how much it must push upward to support the weight of an object. But can you predict what you would see on the dial of a bathroom scale if you stood on it during an elevator ride? Will you see a value greater than your weight when the elevator starts up? What about when the elevator moves upward at a constant speed: will the scale still read more than your weight at rest? Consider the following example.

Example:
What Does the Bathroom Scale Read in an Elevator?
[link] shows a 75.0-kg man (weight of about 165 lb) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate

of 1.20 m/s^2, and (b) if the elevator moves upward at a constant speed of 1 m/s.



(a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale. $\mathbf{T}$ is the tension in the supporting cable, $\mathbf{w}$ is the weight of the person, $\mathbf{w}_\text{s}$ is the weight of the scale, $\mathbf{w}_\text{e}$ is the weight of the elevator, $\mathbf{F}_\text{s}$ is the force of the scale on the person, $\mathbf{F}_\text{p}$ is the force of the person on the scale, $\mathbf{F}_\text{t}$ is the force of the scale on the floor of the elevator, and $\mathbf{N}$ is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting on the designated system of interest—the person.

Strategy

If the scale is accurate, its reading will equal F_p, the magnitude of the force the person exerts downward on it. [link](a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn as in [link](b). Analysis of the free-body diagram using Newton's laws can produce answers to both parts (a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight $\mathbf{w}$ and the upward force of the scale $\mathbf{F}_\text{s}$. According to Newton's third law $\mathbf{F}_\text{p}$ and $\mathbf{F}_\text{s}$ are

equal in magnitude and opposite in direction, so that we need to find F_s in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,
Equation:

$$F_{\text{net}} = \text{ma}.$$

From the free-body diagram we see that $F_{\text{net}} = F_s - w$, so that
Equation:

$$F_s - w = \text{ma}.$$

Solving for F_s gives an equation with only one unknown:
Equation:

$$F_s = \text{ma} + w,$$

or, because $w = mg$, simply
Equation:

$$F_s = \text{ma} + \text{mg}.$$

No assumptions were made about the acceleration, and so this solution should be valid for a variety of accelerations in addition to the ones in this exercise.
Solution for (a)
In this part of the problem, $a = 1.20\ \text{m/s}^2$, so that
Equation:

$$F_s = (75.0\ \text{kg})(1.20\ \text{m/s}^2) + (75.0\ \text{kg})(9.80\ \text{m/s}^2),$$

yielding
Equation:

$$F_s = 825\ \text{N}.$$

Discussion for (a)
This is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:
Equation:

$$\begin{aligned}
F_{\text{net}} &= \text{ma} = 0 = F_s - w \\
F_s &= w = \text{mg} \\
F_s &= (75.0\ \text{kg})(9.80\ \text{m/s}^2) \\
F_s &= 735\ \text{N}.
\end{aligned}$$

So, the scale reading in the elevator is greater than his 735-N (165 lb) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward. Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators.

Solution for (b)

Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because $a = \frac{\Delta v}{\Delta t}$, and $\Delta v = 0$.

Thus,

Equation:

$$F_{\mathrm{s}} = \mathrm{ma} + \mathrm{mg} = 0 + \mathrm{mg}.$$

Now

Equation:

$$F_{\mathrm{s}} = (75.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2}),$$

which gives

Equation:

$$F_{\mathrm{s}} = 735\ \mathrm{N}.$$

Discussion for (b)

The scale reading is 735 N, which equals the person's weight. This will be the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

The solution to the previous example also applies to an elevator accelerating downward, as mentioned. When an elevator accelerates downward, a is negative, and the scale reading is *less* than the weight of the person, until a constant downward velocity is reached, at which time the scale reading again becomes equal to the person's weight. If the elevator is in free-fall and accelerating downward at g, then the scale reading will be zero and the person will *appear* to be weightless.

Integrating Concepts: Newton's Laws of Motion and Kinematics

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton's laws of motion can also be integrated with other concepts that have been discussed previously in this text to

solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters. When approaching problems that involve various types of forces, acceleration, velocity, and/or position, use the following steps to approach the problem:

Problem-Solving Strategy

Step 1. *Identify which physical principles are involved*. Listing the givens and the quantities to be calculated will allow you to identify the principles involved.
Step 2. *Solve the problem using strategies outlined in the text*. If these are available for the specific topic, you should refer to them. You should also refer to the sections of the text that deal with a particular topic. The following worked example illustrates how these strategies are applied to an integrated concept problem.

Example:
What Force Must a Soccer Player Exert to Reach Top Speed?
A soccer player starts from rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What was his average acceleration? (b) What average force did he exert backward on the ground to achieve this acceleration? The player's mass is 70.0 kg, and air resistance is negligible.
Strategy

To solve an *integrated concept problem*, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example considers *acceleration* along a straight line. This is a topic of *kinematics*. Part (b) deals with *force*, a topic of *dynamics* found in this chapter.

The following solutions to each part of the example illustrate how the specific

problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so forth.

Solution for (a)
We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is $\Delta v = 8.00\ \mathrm{m/s}$. We are given the elapsed time, and so $\Delta t = 2.50$ s. The unknown is acceleration, which can be found from its definition:
Equation:

$$a = \frac{\Delta v}{\Delta t}.$$

Substituting the known values yields
Equation:

$$\begin{aligned} a &= \frac{8.00\ \mathrm{m/s}}{2.50\ \mathrm{s}} \\ &= 3.20\ \mathrm{m/s}^2. \end{aligned}$$

Discussion for (a)
This is an attainable acceleration for an athlete in good condition.
Solution for (b)
Here we are asked to find the average force the player exerts backward to achieve this forward acceleration. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes his acceleration. Since we now know the player's acceleration and are given his mass, we can use Newton's second law to find the force exerted. That is,
Equation:

$$F_{\text{net}} = \text{ma}.$$

Substituting the known values of m and a gives
Equation:

$$\begin{aligned} F_{\text{net}} &= (70.0\ \text{kg})(3.20\ \mathrm{m/s}^2) \\ &= 224\ \text{N}. \end{aligned}$$

Discussion for (b)
This is about 50 pounds, a reasonable average force.
This worked example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These strategies are found throughout the text, and many worked examples show how to use them for single topics. You will find these

techniques for integrated concept problems useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

Summary

- Newton's laws of motion can be applied in numerous situations to solve problems of motion.
- Some problems will contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether $F_{\text{net}} = \text{ma}$ or $F_{\text{net}} = 0$.
- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.
- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.

Conceptual Questions

Exercise:

Problem:

To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at g. Why will they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?

Exercise:

Problem:

A cartoon shows the toupee coming off the head of an elevator passenger when the elevator rapidly stops during an upward ride. Can this really happen without the person being tied to the floor of the elevator? Explain your answer.

Problem Exercises

Exercise:

Problem:

A flea jumps by exerting a force of 1.20×10^{-5} N straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of 0.500×10^{-6} N on the flea. Find the direction and magnitude of the acceleration of the flea if its mass is 6.00×10^{-7} kg. Do not neglect the gravitational force.

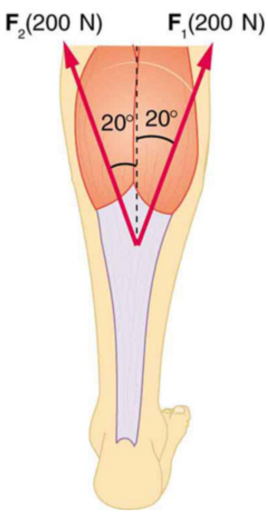
Solution:

10.2 m/s^2, 4.67^o from vertical

Exercise:

Problem:

Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in [link]. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?



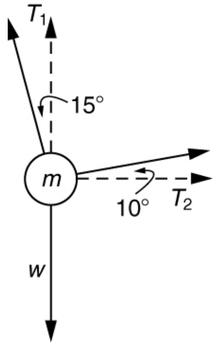
Achilles tendon

Exercise:

Problem:

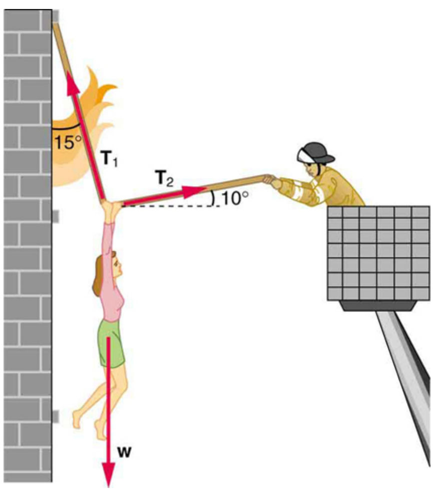
A 76.0-kg person is being pulled away from a burning building as shown in [link]. Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.

Solution:



$$T_1 = 736 \text{ N}$$

$$T_2 = 194 \text{ N}$$



The force $\mathbf{T}_2$ needed to hold steady the person being rescued from the fire is less than her weight and less than the force $\mathbf{T}_1$ in the other rope, since the more

vertical rope supports a greater part of her weight (a vertical force).

Exercise:

Problem:

Integrated Concepts A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow him if he was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)

Exercise:

Problem:

Integrated Concepts When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel?

Solution:

(a) $7.43\ \mathrm{m/s}$

(b) 2.97 m

Exercise:

Problem:

Integrated Concepts A large rocket has a mass of 2.00×10^6 kg at takeoff, and its engines produce a thrust of $3.50 \times 10^7\ \mathrm{N}$. (a) Find its initial acceleration if it takes off vertically. (b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust? (c) In reality, the mass of a rocket decreases significantly as its fuel is consumed. Describe qualitatively how this affects the acceleration and time for this motion.

Exercise:

Problem:

Integrated Concepts A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor. (a) Calculate his velocity when he leaves the floor. (b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in part (a) in a distance of 0.300 m. (c) Calculate the force he exerts on the floor to do this, given that his mass is 110 kg.

Solution:

(a) $4.20\ \text{m/s}$

(b) $29.4\ \text{m/s}^2$

(c) $4.31 \times 10^3\ \text{N}$

Exercise:

Problem:

Integrated Concepts A 2.50-kg fireworks shell is fired straight up from a mortar and reaches a height of 110 m. (a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the shell's velocity when it leaves the mortar. (b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.

Exercise:

Problem:

Integrated Concepts Repeat [link] for a shell fired at an angle 10.0º from the vertical.

Solution:

(a) 47.1 m/s

(b) $2.47 \times 10^3\ \text{m/s}^2$

(c) $6.18 \times 10^3\ \text{N}$. The average force is 252 times the shell's weight.

Exercise:

Problem:

Integrated Concepts An elevator filled with passengers has a mass of 1700 kg. (a) The elevator accelerates upward from rest at a rate of $1.20\ \text{m/s}^2$ for 1.50 s. Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time? (c) The elevator decelerates at a rate of $0.600\ \text{m/s}^2$ for 3.00 s. What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?

Exercise:

Problem:

Unreasonable Results (a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of $0.400\ \text{m/s}^2$ for 50.0 s? (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

Exercise:

Problem:

Unreasonable Results A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

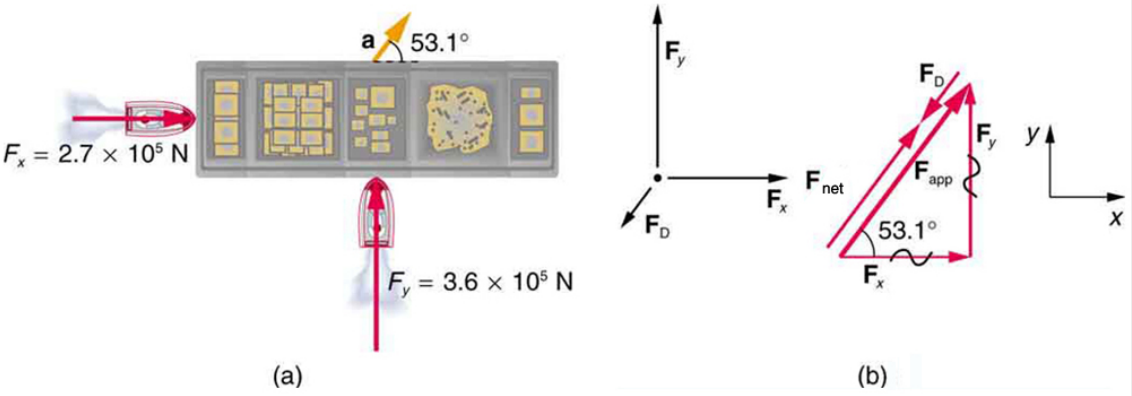
Further Applications of Newton’s Laws of Motion

- Apply problem-solving techniques to solve for quantities in more complex systems of forces.
- Integrate concepts from kinematics to solve problems using Newton's laws of motion.

There are many interesting applications of Newton’s laws of motion, a few more of which are presented in this section. These serve also to illustrate some further subtleties of physics and to help build problem-solving skills.

Example:
Drag Force on a Barge

Suppose two tugboats push on a barge at different angles, as shown in [link]. The first tugboat exerts a force of 2.7×10^5 N in the *x*-direction, and the second tugboat exerts a force of 3.6×10^5 N in the *y*-direction.



(a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the *x*- and *y*-axes are in the same direction as $\mathbf{F}_x$ and $\mathbf{F}_y$. The problem quickly becomes a one-dimensional problem along the direction of $\mathbf{F}_{\text{app}}$, since friction is in the direction opposite to $\mathbf{F}_{\text{app}}$.

If the mass of the barge is 5.0×10^6 kg and its acceleration is observed to be $7.5 \times 10^{-2}\ \text{m/s}^2$ in the direction shown, what is the drag force of the water on the

barge resisting the motion? (Note: drag force is a frictional force exerted by fluids, such as air or water. The drag force opposes the motion of the object.)

Strategy

The directions and magnitudes of acceleration and the applied forces are given in [link]**(a)**. We will define the total force of the tugboats on the barge as $\mathbf{F}_{\text{app}}$ so that:

Equation:

$$\mathbf{F}_{\text{app}}=\mathbf{F}_x+\mathbf{F}_y$$

Since the barge is flat bottomed, the drag of the water $\mathbf{F}_{\text{D}}$ will be in the direction opposite to $\mathbf{F}_{\text{app}}$, as shown in the free-body diagram in [link](b). The system of interest here is the barge, since the forces on *it* are given as well as its acceleration. Our strategy is to find the magnitude and direction of the net applied force $\mathbf{F}_{\text{app}}$, and then apply Newton's second law to solve for the drag force $\mathbf{F}_{\text{D}}$.

Solution

Since $\mathbf{F}_x$ and $\mathbf{F}_y$ are perpendicular, the magnitude and direction of $\mathbf{F}_{\text{app}}$ are easily found. First, the resultant magnitude is given by the Pythagorean theorem:

Equation:

$$\begin{aligned} F_{\text{app}} &= \sqrt{\mathrm{F}_x^2+\mathrm{F}_y^2} \\ F_{\text{app}} &= \sqrt{(2.7\times10^5\ \mathrm{N})^2+(3.6\times10^5\ \mathrm{N})^2} \quad = \quad 4.5\times10^5\ \mathrm{N}. \end{aligned}$$

The angle is given by

Equation:

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{F_y}{F_x}\right) \\ \theta &= \tan^{-1}\left(\frac{3.6\times10^5\ \mathrm{N}}{2.7\times10^5\ \mathrm{N}}\right) = 53^{\mathrm{o}}, \end{aligned}$$

which we know, because of Newton's first law, is the same direction as the acceleration. $\mathbf{F}_{\text{D}}$ is in the opposite direction of $\mathbf{F}_{\text{app}}$, since it acts to slow down the acceleration. Therefore, the net external force is in the same direction as $\mathbf{F}_{\text{app}}$, but its magnitude is slightly less than $\mathbf{F}_{\text{app}}$. The problem is now one-dimensional. From [link]**(b)**, we can see that

Equation:

$$F_{\text{net}}=F_{\text{app}}-F_{\text{D}}.$$

But Newton's second law states that

Equation:

$$F_{\text{net}} = \text{ma}.$$

Thus,

Equation:

$$F_{\text{app}} - F_{\text{D}} = \text{ma}.$$

This can be solved for the magnitude of the drag force of the water F_{D} in terms of known quantities:

Equation:

$$F_{\text{D}} = F_{\text{app}} - \text{ma}.$$

Substituting known values gives

Equation:

$$\text{F}_{\text{D}} = (4.5 \times 10^5\ \text{N}) - (5.0 \times 10^6\ \text{kg})(7.5 \times 10^{-2}\ \text{m/s}^2) = 7.5 \times 10^4\ \text{N}.$$

The direction of $\mathbf{F}_{\text{D}}$ has already been determined to be in the direction opposite to $\mathbf{F}_{\text{app}}$, or at an angle of 53^{o} south of west.

Discussion

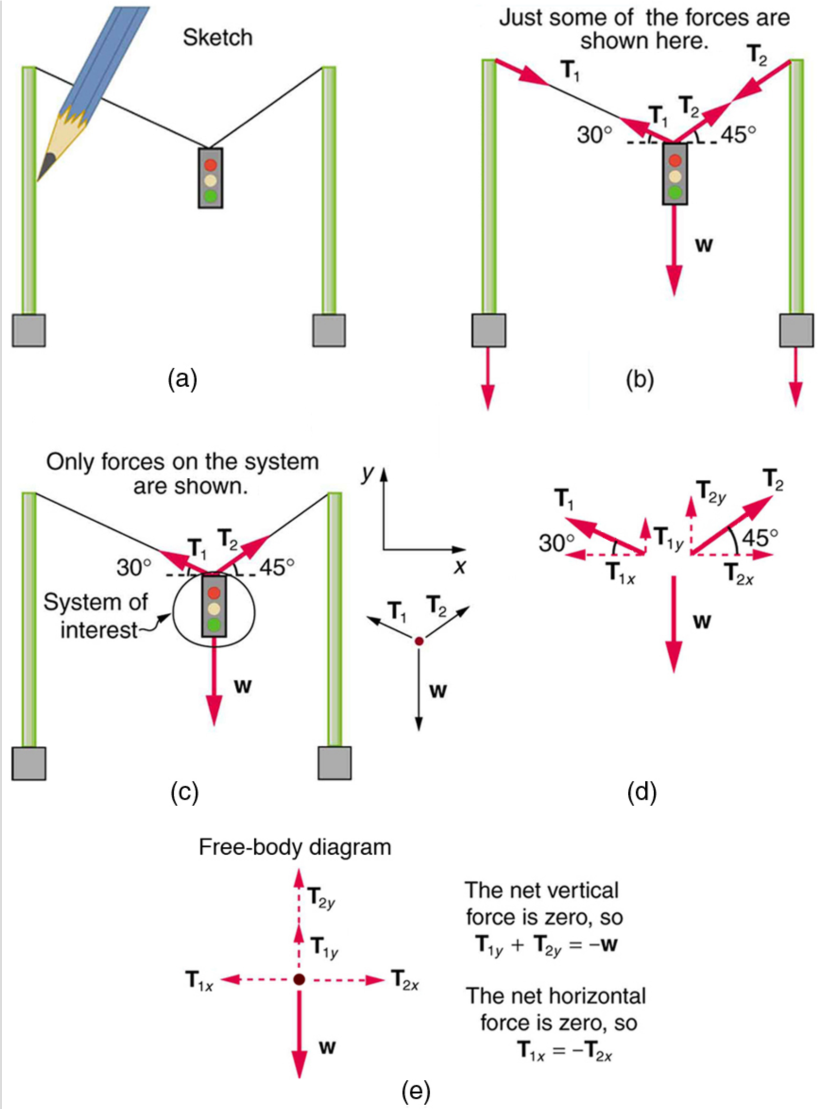
The numbers used in this example are reasonable for a moderately large barge. It is certainly difficult to obtain larger accelerations with tugboats, and small speeds are desirable to avoid running the barge into the docks. Drag is relatively small for a well-designed hull at low speeds, consistent with the answer to this example, where F_{D} is less than 1/600th of the weight of the ship.

In the earlier example of a tightrope walker we noted that the tensions in wires supporting a mass were equal only because the angles on either side were equal. Consider the following example, where the angles are not equal; slightly more trigonometry is involved.

Example:

Different Tensions at Different Angles

Consider the traffic light (mass 15.0 kg) suspended from two wires as shown in [link]. Find the tension in each wire, neglecting the masses of the wires.



A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical (*y*) and horizontal (*x*) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e) The free-body diagram shows the vertical and horizontal forces acting on the traffic light.

Strategy

The system of interest is the traffic light, and its free-body diagram is shown in [link] (c). The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in part (d) of the figure. There are two unknowns in this problem (T_1 and T_2), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

Solution

First consider the horizontal or *x*-axis:

Equation:

$$F_{\text{net}x} = T_{2x} - T_{1x} = 0.$$

Thus, as you might expect,

Equation:

$$T_{1x} = T_{2x}.$$

This gives us the following relationship between T_1 and T_2:

Equation:

$$T_1 \cos\left(30^\circ\right) = T_2 \cos\left(45^\circ\right).$$

Thus,

Equation:

$$T_2 = (1.225)T_1.$$

Note that T_1 and T_2 are not equal in this case, because the angles on either side are not equal. It is reasonable that T_2 ends up being greater than T_1, because it is exerted more vertically than T_1.

Now consider the force components along the vertical or *y*-axis:

Equation:

$$F_{\text{net }y} = T_{1y} + T_{2y} - w = 0.$$

This implies

Equation:

$$T_{1y} + T_{2y} = w.$$

Substituting the expressions for the vertical components gives

Equation:

$$T_1 \sin(30^\circ) + T_2 \sin(45^\circ) = w.$$

There are two unknowns in this equation, but substituting the expression for T_2 in terms of T_1 reduces this to one equation with one unknown:

Equation:

$$T_1(0.500) + (1.225T_1)(0.707) = w = \text{mg},$$

which yields

Equation:

$$(1.366)T_1 = (15.0\ \text{kg})(9.80\ \text{m/s}^2).$$

Solving this last equation gives the magnitude of T_1 to be

Equation:

$$T_1 = 108\ \text{N}.$$

Finally, the magnitude of T_2 is determined using the relationship between them, T_2 = 1.225 T_1, found above. Thus we obtain

Equation:

$$T_2 = 132\ \text{N}.$$

Discussion

Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker).

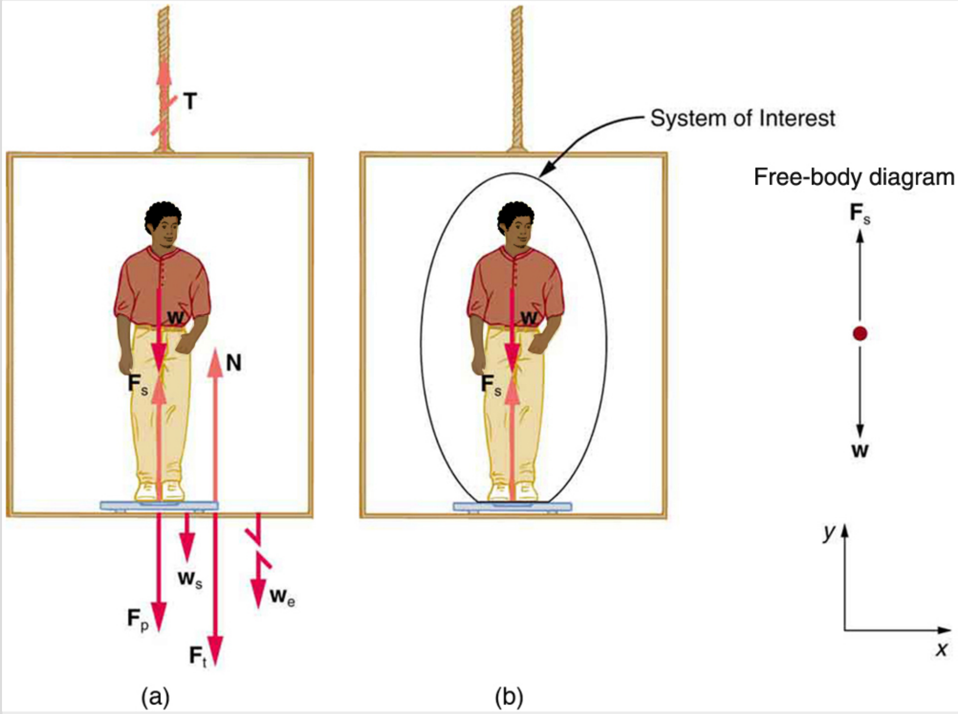
The bathroom scale is an excellent example of a normal force acting on a body. It provides a quantitative reading of how much it must push upward to support the weight of an object. But can you predict what you would see on the dial of a bathroom scale if you stood on it during an elevator ride? Will you see a value greater than your weight when the elevator starts up? What about when the elevator moves upward at a constant speed: will the scale still read more than your weight at rest? Consider the following example.

Example:

What Does the Bathroom Scale Read in an Elevator?

[link] shows a 75.0-kg man (weight of about 165 lb) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate

of 1.20 m/s^2, and (b) if the elevator moves upward at a constant speed of 1 m/s.



(a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale. $\mathbf{T}$ is the tension in the supporting cable, $\mathbf{w}$ is the weight of the person, $\mathbf{w}_\text{s}$ is the weight of the scale, $\mathbf{w}_\text{e}$ is the weight of the elevator, $\mathbf{F}_\text{s}$ is the force of the scale on the person, $\mathbf{F}_\text{p}$ is the force of the person on the scale, $\mathbf{F}_\text{t}$ is the force of the scale on the floor of the elevator, and $\mathbf{N}$ is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting on the designated system of interest—the person.

Strategy

If the scale is accurate, its reading will equal F_p, the magnitude of the force the person exerts downward on it. [link](a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn as in [link](b). Analysis of the free-body diagram using Newton's laws can produce answers to both parts (a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight $\mathbf{w}$ and the upward force of the scale $\mathbf{F}_\text{s}$. According to Newton's third law $\mathbf{F}_\text{p}$ and $\mathbf{F}_\text{s}$ are

equal in magnitude and opposite in direction, so that we need to find F_s in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,
Equation:

$$F_{net} = ma.$$

From the free-body diagram we see that $F_{net} = F_s - w$, so that
Equation:

$$F_s - w = ma.$$

Solving for F_s gives an equation with only one unknown:
Equation:

$$F_s = ma + w,$$

or, because $w = mg$, simply
Equation:

$$F_s = ma + mg.$$

No assumptions were made about the acceleration, and so this solution should be valid for a variety of accelerations in addition to the ones in this exercise.
Solution for (a)
In this part of the problem, $a = 1.20\ \text{m/s}^2$, so that
Equation:

$$F_s = (75.0\ \text{kg})(1.20\ \text{m/s}^2) + (75.0\ \text{kg})(9.80\ \text{m/s}^2),$$

yielding
Equation:

$$F_s = 825\ \text{N}.$$

Discussion for (a)
This is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:
Equation:

$$\begin{aligned} F_{net} &= ma = 0 = F_s - w \\ F_s &= w = mg \\ F_s &= (75.0\ \text{kg})(9.80\ \text{m/s}^2) \\ F_s &= 735\ \text{N}. \end{aligned}$$

So, the scale reading in the elevator is greater than his 735-N (165 lb) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward. Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators.

Solution for (b)

Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because $a = \frac{\Delta v}{\Delta t}$, and $\Delta v = 0$.

Thus,

Equation:

$$F_\text{s} = \text{ma} + \text{mg} = 0 + \text{mg}.$$

Now

Equation:

$$F_\text{s} = (75.0\ \text{kg})(9.80\ \text{m/s}^2),$$

which gives

Equation:

$$F_\text{s} = 735\ \text{N}.$$

Discussion for (b)

The scale reading is 735 N, which equals the person's weight. This will be the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

The solution to the previous example also applies to an elevator accelerating downward, as mentioned. When an elevator accelerates downward, a is negative, and the scale reading is *less* than the weight of the person, until a constant downward velocity is reached, at which time the scale reading again becomes equal to the person's weight. If the elevator is in free-fall and accelerating downward at g, then the scale reading will be zero and the person will *appear* to be weightless.

Integrating Concepts: Newton's Laws of Motion and Kinematics

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton's laws of motion can also be integrated with other concepts that have been discussed previously in this text to

solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters. When approaching problems that involve various types of forces, acceleration, velocity, and/or position, use the following steps to approach the problem:

Problem-Solving Strategy

Step 1. *Identify which physical principles are involved*. Listing the givens and the quantities to be calculated will allow you to identify the principles involved.
Step 2. *Solve the problem using strategies outlined in the text*. If these are available for the specific topic, you should refer to them. You should also refer to the sections of the text that deal with a particular topic. The following worked example illustrates how these strategies are applied to an integrated concept problem.

Example:
What Force Must a Soccer Player Exert to Reach Top Speed?
A soccer player starts from rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What was his average acceleration? (b) What average force did he exert backward on the ground to achieve this acceleration? The player’s mass is 70.0 kg, and air resistance is negligible.
Strategy

To solve an *integrated concept problem*, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example considers *acceleration* along a straight line. This is a topic of *kinematics*. Part (b) deals with *force*, a topic of *dynamics* found in this chapter.

The following solutions to each part of the example illustrate how the specific

problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so forth.

Solution for (a)
We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is $\Delta v = 8.00\ \mathrm{m/s}$. We are given the elapsed time, and so $\Delta t = 2.50\ \mathrm{s}$. The unknown is acceleration, which can be found from its definition:
Equation:

$$a = \frac{\Delta v}{\Delta t}.$$

Substituting the known values yields
Equation:

$$\begin{aligned} a &= \frac{8.00\ \mathrm{m/s}}{2.50\ \mathrm{s}} \\ &= 3.20\ \mathrm{m/s}^2. \end{aligned}$$

Discussion for (a)
This is an attainable acceleration for an athlete in good condition.
Solution for (b)
Here we are asked to find the average force the player exerts backward to achieve this forward acceleration. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes his acceleration. Since we now know the player’s acceleration and are given his mass, we can use Newton’s second law to find the force exerted. That is,
Equation:

$$F_{\mathrm{net}} = \mathrm{ma}.$$

Substituting the known values of m and a gives
Equation:

$$\begin{aligned} F_{\mathrm{net}} &= (70.0\ \mathrm{kg})(3.20\ \mathrm{m/s}^2) \\ &= 224\ \mathrm{N}. \end{aligned}$$

Discussion for (b)
This is about 50 pounds, a reasonable average force.
This worked example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These strategies are found throughout the text, and many worked examples show how to use them for single topics. You will find these

techniques for integrated concept problems useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

Summary

- Newton's laws of motion can be applied in numerous situations to solve problems of motion.
- Some problems will contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether $F_{\text{net}} = \text{ma}$ or $F_{\text{net}} = 0$.
- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.
- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.

Conceptual Questions

Exercise:

Problem:

To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at g. Why will they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?

Exercise:

Problem:

A cartoon shows the toupee coming off the head of an elevator passenger when the elevator rapidly stops during an upward ride. Can this really happen without the person being tied to the floor of the elevator? Explain your answer.

Problem Exercises

Exercise:

Problem:

A flea jumps by exerting a force of 1.20×10^{-5} N straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of 0.500×10^{-6} N on the flea. Find the direction and magnitude of the acceleration of the flea if its mass is 6.00×10^{-7} kg. Do not neglect the gravitational force.

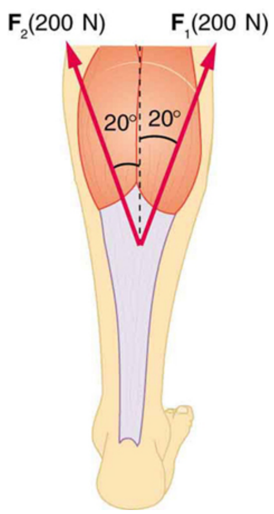
Solution:

10.2 m/s^2, 4.67° from vertical

Exercise:

Problem:

Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in [link]. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?



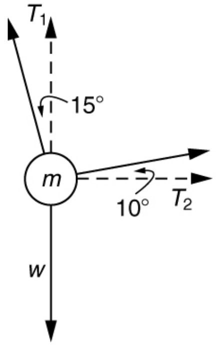
Achilles tendon

Exercise:

Problem:

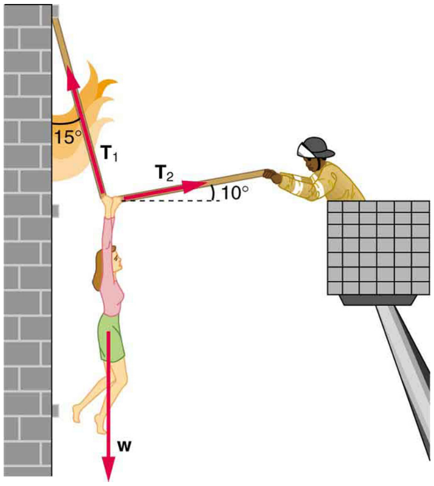
A 76.0-kg person is being pulled away from a burning building as shown in [link]. Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.

Solution:



$$T_1 = 736\ \text{N}$$

$$T_2 = 194\ \text{N}$$



The force $\mathbf{T}_2$ needed to hold steady the person being rescued from the fire is less than her weight and less than the force $\mathbf{T}_1$ in the other rope, since the more

vertical rope supports a
greater part of her weight
(a vertical force).

Exercise:

Problem:

Integrated Concepts A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow him if he was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)

Exercise:

Problem:

Integrated Concepts When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel?

Solution:

(a) 7.43 m/s

(b) 2.97 m

Exercise:

Problem:

Integrated Concepts A large rocket has a mass of 2.00×10^6 kg at takeoff, and its engines produce a thrust of 3.50×10^7 N. (a) Find its initial acceleration if it takes off vertically. (b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust? (c) In reality, the mass of a rocket decreases significantly as its fuel is consumed. Describe qualitatively how this affects the acceleration and time for this motion.

Exercise:

Problem:

Integrated Concepts A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor. (a) Calculate his velocity when he leaves the floor. (b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in part (a) in a distance of 0.300 m. (c) Calculate the force he exerts on the floor to do this, given that his mass is 110 kg.

Solution:

(a) 4.20 m/s

(b) 29.4 m/s^2

(c) $4.31 \times 10^3 \text{ N}$

Exercise:

Problem:

Integrated Concepts A 2.50-kg fireworks shell is fired straight up from a mortar and reaches a height of 110 m. (a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the shell's velocity when it leaves the mortar. (b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.

Exercise:

Problem:

Integrated Concepts Repeat [link] for a shell fired at an angle 10.0º from the vertical.

Solution:

(a) 47.1 m/s

(b) $2.47 \times 10^3 \text{ m/s}^2$

(c) $6.18 \times 10^3 \text{ N}$. The average force is 252 times the shell's weight.

Exercise:

Problem:

Integrated Concepts An elevator filled with passengers has a mass of 1700 kg. (a) The elevator accelerates upward from rest at a rate of 1.20 m/s^2 for 1.50 s. Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time? (c) The elevator decelerates at a rate of 0.600 m/s^2 for 3.00 s. What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?

Exercise:

Problem:

Unreasonable Results (a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of 0.400 m/s^2 for 50.0 s? (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

Exercise:

Problem:

Unreasonable Results A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

Introduction
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This section is also available as a video at https://www.youtube.com/watch?v=ancMsBFzLis

In this chapter, we will looking at the different kinds of forces that we will be talking about in this class. If you look around you, it may seem like there's a huge number of forces in the world around us. Frictions, pushes pulls, air resistance, whatever force causes you to float, these different forces. The goal of this chapter is to acquaint you with the different types of forces that we will be dealing with explicitly in this class.

Fundamentally, there are only four forces, so all the various forces that we see around us are, at the microscopic level, a manifestation of one of these four fundamental forces. In order of strength, the four fundamental forces are the strong nuclear force, which is responsible for holding the protons and neutrons in nuclei together, the electricity and magnetism forces, which you may have some familiarity with from the idea that opposite charges attract, like charges repel, and from playing with magnets, there is also the weak nuclear force which is responsible for radioactive decay, and then the weakest of the four fundamental forces is gravity, which you have some experience with as its the force that holds you to the earth, and holds the earth in orbit around the Sun.

We will not be exploring the strong nuclear force and the weak nuclear force at all in this class. While these forces are very important, their range is limited to sizes smaller than an atomic nucleus, and so don't have visible measurable effects at our everyday scales. Gravity on the other hand, we will talk about in some level of detail. Electricity and magnetism, on the other hand, is primarily dealt with in Physics 132. However, I do you expect you to have the basic understanding that opposite charges attract and like

charges repel, because this is fundamentally the origin of the non-fundamental forces that we will discuss in this class.

So, what non-fundamental forces will we discuss in this class? Non-fundamental forces are forces that at the microscopic scale can be explained in terms of electrical forces, but at the macroscopic scale, we just average over all the atoms and call it a new type of force. For example, the normal force the normal force is probably best understood by setting a book on top of a table. The gravitational force pulls the book down; why doesn't the book just fall through the table? Well, there is a normal force from the table on the book to counter this force of gravity. At the microscopic scale, this normal force arises from the propulsion of electrons in the book to the electrons within the table. So, at the microscopic scale, this force is electrical, however, at our macroscopic scale that we deal with in our everyday world, we're averaging over these different atoms and just calling their net effect a normal force. One characteristic of the normal force is that it's perpendicular. It's always perpendicular. In fact, the word "normal" means perpendicular in mathematic-ese, so in mathematics, the word normal and the word perpendicular are just synonyms. This can help you remember the directions of the normal force. Another non-fundamental force we will discuss in this class is tension. Tension really arises when you start to have ropes and chains and that kind of a thing.

Consider a box hanging from a rope. Again, the force of gravity is pulling the box down. What keeps the box from falling? There is a tension in the rope that is countering the weight of the box holding it up. Again, at the microscopic scale, the tension force, arises from electricity, as the atomic bonds which are electrical in nature between one molecule of rope and the next are responsible for this force of tension. We'll also discuss forces involved with springs such as big metal coils that you might have had some experience with.

When I compress a spring, the spring exerts a force back outward as it tries to re-expand. You can imagine your hand compressing the spring, the spring would be pushing outward in the direction of this blue arrow when it is compressed. Conversely, if I stretch the spring, the direction of the spring

on your hand would then be in the opposite direction, as the spring tries to pull itself back to its rest length.

The final set of non-fundamental forces we will discuss are frictional forces. These are the forces that come when you have rough surfaces in contact and are fundamentally electrical, and arise from Van der Waals interactions in hydrogen bonds between surfaces. There are two different kinds of friction. One is static friction, this is what happens when objects are not moving relative to each other, and then there is kinetic friction, which occurs when objects are sliding past each other. The directions of frictional forces can sometimes be somewhat tricky, and we'll have a lab in class to directly deal with them.

In summary, there are only four fundamental forces: the strong nuclear force, the electric and magnetic forces, weak nuclear force, and gravity. The only fundamental force will deal with in this glass is the weakest of the four, the gravitational force. We will also deal with five non-fundamental forces that are just electrical in nature. The normal force, which is what prevents objects from passing through to each other. It's due to electrical repulsion and is always perpendicular to the surfaces between objects. We'll also talk about tension forces, which come into play when you're dealing with ropes, chains, and the like. These are due to molecular bonds and therefore also electrical, and the direction of tension forces is always along the direction of the rope. We'll talk about spring forces, which of course deal, we'll talk about spring forces which of course come into play when we're talking about springs, and the direction of spring forces depends upon if the spring is either being stretched or compressed. Finally, we'll talk about friction forces, which at the microscopic level are due to van der Waals interactions in hydrogen bonds. We'll talk about both kinds of friction. Static friction, which occurs when objects are not moving relative to each other, this is the force you need to overcome to get an object to move, and we'll discuss kinetic friction which is the friction that occurs when objects are sliding past each other. This is the force that you need to overcome to keep an object moving across the rough surface.

The Fundamental Forces

- Understand the four basic forces that underlie the processes in nature.

One of the most remarkable simplifications in physics is that only four distinct forces account for all known phenomena. In fact, nearly all of the forces we experience directly are due to only one basic force, called the electromagnetic force. (The gravitational force is the only force we experience directly that is not electromagnetic.) This is a tremendous simplification of the myriad of *apparently* different forces we can list, only a few of which were discussed in the previous section. As we will see, the basic forces are all thought to act through the exchange of microscopic carrier particles, and the characteristics of the basic forces are determined by the types of particles exchanged. Action at a distance, such as the gravitational force of Earth on the Moon, is explained by the existence of a **force field** rather than by "physical contact."

The *four basic forces* are the gravitational force, the electromagnetic force, the weak nuclear force, and the strong nuclear force. Their properties are summarized in [link]. Since the weak and strong nuclear forces act over an extremely short range, the size of a nucleus or less, we do not experience them directly, although they are crucial to the very structure of matter. These forces determine which nuclei are stable and which decay, and they are the basis of the release of energy in certain nuclear reactions. Nuclear forces determine not only the stability of nuclei, but also the relative abundance of elements in nature. The properties of the nucleus of an atom determine the number of electrons it has and, thus, indirectly determine the chemistry of the atom. More will be said of all of these topics in later chapters.

Note:
Concept Connections: The Four Basic Forces
The four basic forces will be encountered in more detail as you progress through the text. The gravitational force is defined in Uniform Circular Motion and Gravitation, electric force in Electric Charge and Electric Field, magnetic force in Magnetism, and nuclear forces in Radioactivity and Nuclear Physics. On a macroscopic scale, electromagnetism and gravity are the basis for all forces. The nuclear forces are vital to the substructure of matter, but they are not directly experienced on the macroscopic scale.

Force	Approximate Relative Strengths	Range	Attraction/Repulsion	Carrier Particle
Gravitational	10^{-38}	∞	attractive only	Graviton

Force	Approximate Relative Strengths	Range	Attraction/Repulsion	Carrier Particle
Electromagnetic	10^{-2}	∞	attractive and repulsive	Photon
Weak nuclear	10^{-13}	$< 10^{-18} m$	attractive and repulsive	W^{+}, W^{-}, Z^{0}
Strong nuclear	1	$< 10^{-15} m$	attractive and repulsive	gluons

Properties of the Four Basic Forces[footnote]
The graviton is a proposed particle, though it has not yet been observed by scientists. See the discussion of gravitational waves later in this section. The particles W^{+}, W^{-}, and Z^{0} are called vector bosons; these were predicted by theory and first observed in 1983. There are eight types of gluons proposed by scientists, and their existence is indicated by meson exchange in the nuclei of atoms.

The gravitational force is surprisingly weak—it is only because gravity is always attractive that we notice it at all. Our weight is the gravitational force due to the *entire* Earth acting on us. On the very large scale, as in astronomical systems, the gravitational force is the dominant force determining the motions of moons, planets, stars, and galaxies. The gravitational force also affects the nature of space and time. As we shall see later in the study of general relativity, space is curved in the vicinity of very massive bodies, such as the Sun, and time actually slows down near massive bodies.

Electromagnetic forces can be either attractive or repulsive. They are long-range forces, which act over extremely large distances, and they nearly cancel for macroscopic objects. (Remember that it is the *net* external force that is important.) If they did not cancel, electromagnetic forces would completely overwhelm the gravitational force. The electromagnetic force is a combination of electrical forces (such as those that cause static electricity) and magnetic forces (such as those that affect a compass needle). These two forces were thought to be quite distinct until early in the 19th century, when scientists began to discover that they are different manifestations of the same force. This discovery is a classical case of the *unification of forces*. Similarly, friction, tension, and all of the other classes of forces we experience directly (except gravity, of course) are due to electromagnetic interactions of atoms and molecules. It is still convenient to consider these forces separately in specific applications, however, because of the ways they manifest themselves.

Note:
Concept Connections: Unifying Forces

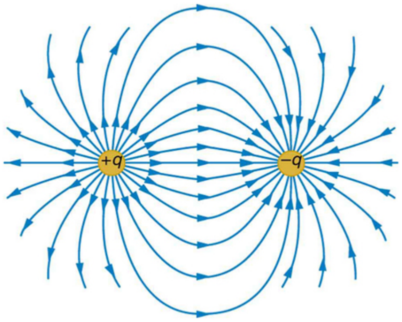
Attempts to unify the four basic forces are discussed in relation to elementary particles later in this text. By “unify” we mean finding connections between the forces that show that they are different manifestations of a single force. Even if such unification is achieved, the forces will retain their separate characteristics on the macroscopic scale and may be identical only under extreme conditions such as those existing in the early universe.

Physicists are now exploring whether the four basic forces are in some way related. Attempts to unify all forces into one come under the rubric of Grand Unified Theories (GUTs), with which there has been some success in recent years. It is now known that under conditions of extremely high density and temperature, such as existed in the early universe, the electromagnetic and weak nuclear forces are indistinguishable. They can now be considered to be different manifestations of one force, called the *electroweak* force. So the list of four has been reduced in a sense to only three. Further progress in unifying all forces is proving difficult—especially the inclusion of the gravitational force, which has the special characteristics of affecting the space and time in which the other forces exist.

While the unification of forces will not affect how we discuss forces in this text, it is fascinating that such underlying simplicity exists in the face of the overt complexity of the universe. There is no reason that nature must be simple—it simply is.

Action at a Distance: Concept of a Field

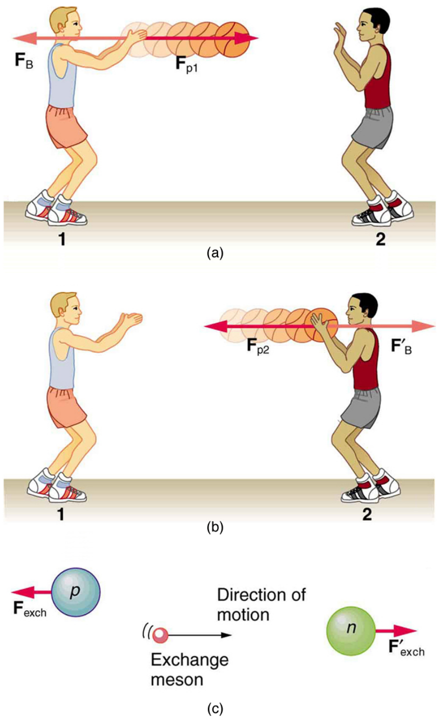
All forces act at a distance. This is obvious for the gravitational force. Earth and the Moon, for example, interact without coming into contact. It is also true for all other forces. Friction, for example, is an electromagnetic force between atoms that may not actually touch. What is it that carries forces between objects? One way to answer this question is to imagine that a **force field** surrounds whatever object creates the force. A second object (often called a *test object*) placed in this field will experience a force that is a function of location and other variables. The field itself is the “thing” that carries the force from one object to another. The field is defined so as to be a characteristic of the object creating it; the field does not depend on the test object placed in it. Earth’s gravitational field, for example, is a function of the mass of Earth and the distance from its center, independent of the presence of other masses. The concept of a field is useful because equations can be written for force fields surrounding objects (for gravity, this yields $w = \text{mg}$ at Earth’s surface), and motions can be calculated from these equations. (See [link].)



The electric force field between a positively charged particle and a negatively charged particle. When a positive test charge is placed in the field, the charge will experience a force in the direction of the force field lines.

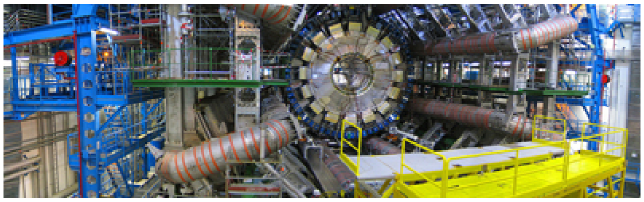
Note:
Concept Connections: Force Fields
The concept of a *force field* is also used in connection with electric charge and is presented in Electric Charge and Electric Field. It is also a useful idea for all the basic forces, as will be seen in Particle Physics. Fields help us to visualize forces and how they are transmitted, as well as to describe them with precision and to link forces with subatomic carrier particles.

The field concept has been applied very successfully; we can calculate motions and describe nature to high precision using field equations. As useful as the field concept is, however, it leaves unanswered the question of what carries the force. It has been proposed in recent decades, starting in 1935 with Hideki Yukawa's (1907–1981) work on the strong nuclear force, that all forces are transmitted by the exchange of elementary particles. We can visualize particle exchange as analogous to macroscopic phenomena such as two people passing a basketball back and forth, thereby exerting a repulsive force without touching one another. (See [link].)



The exchange of masses resulting in repulsive forces. (a) The person throwing the basketball exerts a force $\mathbf{F}_{\text{p1}}$ on it toward the other person and feels a reaction force $\mathbf{F}_{\text{B}}$ away from the second person. (b) The person catching the basketball exerts a force $\mathbf{F}_{\text{p2}}$ on it to stop the ball and feels a reaction force $\mathbf{F'}_{\text{B}}$ away from the first person. (c) The analogous exchange of a meson between a proton and a neutron carries the strong nuclear forces $\mathbf{F}_{\text{exch}}$ and $\mathbf{F'}_{\text{exch}}$ between them. An attractive force can also be exerted by the exchange of a mass—if person 2 pulled the basketball away from the first person as he tried to retain it, then the force between them would be attractive.

This idea of particle exchange deepens rather than contradicts field concepts. It is more satisfying philosophically to think of something physical actually moving between objects acting at a distance. [link] lists the exchange or **carrier particles**, both observed and proposed, that carry the four forces. But the real fruit of the particle-exchange proposal is that searches for Yukawa's proposed particle found it *and* a number of others that were completely unexpected, stimulating yet more research. All of this research eventually led to the proposal of quarks as the underlying substructure of matter, which is a basic tenet of GUTs. If successful, these theories will explain not only forces, but also the structure of matter itself. Yet physics is an experimental science, so the test of these theories must lie in the domain of the real world. As of this writing, scientists at the CERN laboratory in Switzerland are starting to test these theories using the world's largest particle accelerator: the Large Hadron Collider. This accelerator (27 km in circumference) allows two high-energy proton beams, traveling in opposite directions, to collide. An energy of 14 trillion electron volts will be available. It is anticipated that some new particles, possibly force carrier particles, will be found. (See [link].) One of the force carriers of high interest that researchers hope to detect is the Higgs boson. The observation of its properties might tell us why different particles have different masses.



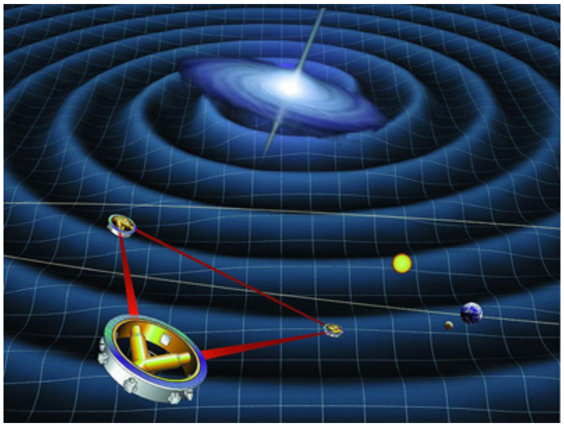
The world's largest particle accelerator spans the border between Switzerland and France. Two beams, traveling in opposite directions close to the speed of light, collide in a tube similar to the central tube shown here. External magnets determine the beam's path. Special detectors will analyze particles created in these collisions. Questions as broad as what is the origin of mass and what was matter like the first few seconds of our universe will be explored. This accelerator began preliminary operation in 2008. (credit: Frank Hommes)

Tiny particles also have wave-like behavior, something we will explore more in a later chapter. To better understand force-carrier particles from another perspective, let us consider gravity. The search for gravitational waves has been going on for a number of years. Almost 100 years

ago, Einstein predicted the existence of these waves as part of his general theory of relativity. Gravitational waves are created during the collision of massive stars, in black holes, or in supernova explosions—like shock waves. These gravitational waves will travel through space from such sites much like a pebble dropped into a pond sends out ripples—except these waves move at the speed of light. A detector apparatus has been built in the U.S., consisting of two large installations nearly 3000 km apart—one in Washington state and one in Louisiana! The facility is called the Laser Interferometer Gravitational-Wave Observatory (LIGO). Each installation is designed to use optical lasers to examine any slight shift in the relative positions of two masses due to the effect of gravity waves. The two sites allow simultaneous measurements of these small effects to be separated from other natural phenomena, such as earthquakes. Initial operation of the detectors began in 2002, and work is proceeding on increasing their sensitivity. Similar installations have been built in Italy (VIRGO), Germany (GEO600), and Japan (TAMA300) to provide a worldwide network of gravitational wave detectors.

International collaboration in this area is moving into space with the joint EU/US project LISA (Laser Interferometer Space Antenna). Earthquakes and other Earthly noises will be no problem for these monitoring spacecraft. LISA will complement LIGO by looking at much more massive black holes through the observation of gravitational-wave sources emitting much larger wavelengths. Three satellites will be placed in space above Earth in an equilateral triangle (with 5,000,000-km sides) ([link]). The system will measure the relative positions of each satellite to detect passing gravitational waves. Accuracy to within 10% of the size of an atom will be needed to detect any waves. The launch of this project might be as early as 2018.

“I’m sure LIGO will tell us something about the universe that we didn’t know before. The history of science tells us that any time you go where you haven’t been before, you usually find something that really shakes the scientific paradigms of the day. Whether gravitational wave astrophysics will do that, only time will tell.” —David Reitze, LIGO Input Optics Manager, University of Florida



Space-based future experiments for the measurement of gravitational waves. Shown here is a drawing of

LISA's orbit. Each satellite of LISA will consist of a laser source and a mass. The lasers will transmit a signal to measure the distance between each satellite's test mass. The relative motion of these masses will provide information about passing gravitational waves. (credit: NASA)

The ideas presented in this section are but a glimpse into topics of modern physics that will be covered in much greater depth in later chapters.

Summary

- The various types of forces that are categorized for use in many applications are all manifestations of the *four basic forces* in nature.
- The properties of these forces are summarized in [link].
- Everything we experience directly without sensitive instruments is due to either electromagnetic forces or gravitational forces. The nuclear forces are responsible for the submicroscopic structure of matter, but they are not directly sensed because of their short ranges. Attempts are being made to show all four forces are different manifestations of a single unified force.
- A force field surrounds an object creating a force and is the carrier of that force.

Conceptual Questions

Exercise:

Problem:

Explain, in terms of the properties of the four basic forces, why people notice the gravitational force acting on their bodies if it is such a comparatively weak force.

Exercise:

Problem:

What is the dominant force between astronomical objects? Why are the other three basic forces less significant over these very large distances?

Exercise:

Problem:

Give a detailed example of how the exchange of a particle can result in an *attractive* force. (For example, consider one child pulling a toy out of the hands of another.)

Problem Exercises

Exercise:

Problem:

(a) What is the strength of the weak nuclear force relative to the strong nuclear force? (b) What is the strength of the weak nuclear force relative to the electromagnetic force? Since the weak nuclear force acts at only very short distances, such as inside nuclei, where the strong and electromagnetic forces also act, it might seem surprising that we have any knowledge of it at all. We have such knowledge because the weak nuclear force is responsible for beta decay, a type of nuclear decay not explained by other forces.

Solution:

(a) 1×10^{-13}

(b) 1×10^{-11}

Exercise:

Problem:

(a) What is the ratio of the strength of the gravitational force to that of the strong nuclear force? (b) What is the ratio of the strength of the gravitational force to that of the weak nuclear force? (c) What is the ratio of the strength of the gravitational force to that of the electromagnetic force? What do your answers imply about the influence of the gravitational force on atomic nuclei?

Exercise:

Problem:

What is the ratio of the strength of the strong nuclear force to that of the electromagnetic force? Based on this ratio, you might expect that the strong force dominates the nucleus, which is true for small nuclei. Large nuclei, however, have sizes greater than the range of the strong nuclear force. At these sizes, the electromagnetic force begins to affect nuclear stability. These facts will be used to explain nuclear fusion and fission later in this text.

Solution:

10^{2}

Glossary

carrier particle
: a fundamental particle of nature that is surrounded by a characteristic force field; photons are carrier particles of the electromagnetic force

force field

a region in which a test particle will experience a force

Weight and Gravity
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This section is also available as a video, available here: https://www.youtube.com/watch?v=5RiNj5IiTbg

Gravity is one of the fundamental forces. We're going to explain gravitational interactions in terms of the idea of the gravitational field, which we indicate by the little letter g. We'll discover that this field is a vector, hence the little vector symbol above the variable. We're going to describe when the so-called flat earth gravity is a valid approximation.

So, let's think a little bit about the history of physics' understanding of the force of gravity. The first real description of the force of gravity comes from Isaac Newton in 1687. Isaac Newton was the first person to think about the fact that the same force that causes an apple to fall from a tree keeps the moon in orbit around the Earth. Both are consequences of the force of gravity. But, you might say to yourself, the apple falls straight down, while everyone knows that the moon goes around and around and around. These seem like fundamentally different motions.

Here's an applet that can help us think about gravity like Isaac Newton did: http://waowen.screaming.net/revision/force&motion/ncananim.htm (if you cannot get the applet to work, the YouTube version of this section has a run through of this applet). Play around with the velocities; try launching the ball at 2000 m/s, 3000 m/s, and 4500 m/s. For the 2000 m/s and 3000 m/s launch, the cannonball shoots out, goes some distance, and then falls to the earth. However, the cannonball at 4500 m/s acts differently. The cannonball never actually hits the earth. It keeps falling around and around and around and around, without ever hitting, and this is the crux of what an orbit is. It's falling and then missing. The reason you miss is because the earth falls away faster than you fall towards the center, and this was Isaac Newton's

realization, that the same force of attraction that pulls an apple to the ground holds the moon in orbit around the earth.

This was a very revolutionary idea for the time because back in the 17th century it was thought that different physical laws operated on earth then operated in “the heavens”, as they called it at that time. So, apple falling and moon orbiting related by the fundamental idea of gravity. This is one of the most powerful and first examples of a single fundamental idea explaining a variety of different phenomena, and really shows you the power of fundamental ideas in physics.

So, let’s stop and think for a second. How does the moon, or if you prefer, the apple know that the earth is there? I mean, the moon is very far away from the earth. It’s not touching the earth; how does the moon know that the earth is there? Well, Isaac Newton himself could not come up with a particularly good answer to this question. He called it “action at a distance” and sort of left it at that. Now, the way we modern physicists envision this is we say that the earth generates what’s known as a gravitational field, and this gravitational field is an invisible field that extends out from the earth in all directions. This is the gravitational field that we indicate in this class by little g, and it has a direction so it’s a vector, hence the little vector symbol above the g. The moon does touch the gravitational field, because this gravitational field, these are just some sample lines, the gravitational field goes everywhere, so the moon does touch the gravitational field of the earth, and it responds to this gravitational field by feeling a force, and the magnitude of the moon’s force, or the magnitude of the force from the earth on the moon, is the magnitude of the gravitational field from the earth at the spot of the moon multiplied by the mass of the moon.

This is the fundamental idea of the field. This field concept will be used at great more length in Physics 132 in the context of the electric field. Looking at this definition of force in terms of gravitational field, we can see the units of gravitational field. The units of force are newtons, the units of mass are kilograms, and therefore the units of the gravitational field must be newtons over kilograms, or people will say it newtons per kilogram.

So, let’s review the fundamental features of this gravitational field. Every object with mass, not just planets but every object with mass, including

yourself, generates a gravitational field, little g. Now, planets are the only things that generate big enough gravitational fields to matter, but everything generates a gravitational field. Every object with mass interacts with all the fields around it by feeling a force.

So, in the case of our apple the earth generates a gravitational field down, and the apple interacts with that field by feeling a force towards the Earth. You might ask yourself, “well, doesn’t the apple generate its own field?”. Yes, the apple does generate its own field, albeit a very tiny one, so the apple will also generate a very tiny field towards it, and the earth will respond to that tiny field by feeling a force upwards. We’ll talk a little bit more about this seeming paradox, we don’t see the earth move, in class, but it is true. So, every object interacts with all the other fields by feeling a force, m times g. It’s important to keep in mind that objects don’t or interact with their own field, they only interact with the surrounding fields. So, the apple interacts with the field of the earth and the earth interacts with the field from the apple. The earth doesn’t interact with its own field. We’ve already talked about the fact that the units of g are newtons per kilogram, and the consequence of this is that every object in the universe with mass attracts each other. So, any two objects with mass in the entire universe attract each other, which might lead you to the question, “why doesn’t the whole universe just collapse?”. If everything with mass is attracting each other, it seems like everything we just fall into one big giant heap. Well the answer is that the field, and therefore the force, since the force is equal to the mass of an object times the field, gets smaller with distance. So, if the field gets smaller, the force will get smaller, and this gets smaller with distance from the center of the object. That’s the relevant quantity not distance from the surface, but distance from the center. This has important implications for our class.

So, how are we going to deal with gravity in this class? Remember. the gravitational field gets weaker as the distance from the center of the object. in our case. the earth. because we’re all on the earth. increases. In this course, we’re going to be dealing with everyday heights that we all can experience. These are all very small compared to the radius of the Earth. The radius of the Earth is 10^6 meters: 6 million meters. Even if you were to go to the top of Mount Everest, which is the highest mountain on Earth,

as you probably, know that’s still an only an extra 8,000 meters, and you’ve only increased your distance from the center of the earth by a very tiny amount. Thus, for this course we are always essentially the radius of the earth from the center. This is called the “flat-earth approximation” now this might seem like a very strange idea; you’re in a university physics class and we’re talking about the earth being flat. Well, we’re not really. We’re making an approximation at the earth is flat, and the approximation is in other words that the earth is very big compared to anything else we’re dealing with. Even compared to Mount Everest the earth is huge, and so we can treat it as very big, in which case it is essentially flat. We don’t have to worry about the fact that the earth is round so thus it’s called the “flat-earth approximation”. Now, if you were to go to, say, the moon, which is many times further away than the size of the earth, this approximation would no longer be valid, but if you’re close to the surface of the earth and this approximation is good, then the gravitational field is going to be essentially constant. We’re only moving very tiny amounts relative to the radius of the earth, so the gravitational field as far as we are ever going to experience is not really going to change. It’s going to be constant. This constant value has been measured to be 9.8 newtons per kilogram. Therefore, in this class, we will say that the force of gravity from the earth on an object, whatever it is, and forces are vectors, will be the mass of the object, say an apple, times g, where this g is 9.8 newtons per kilogram.

We will also occasionally speak of this force of gravity as the weight force. This is just equivalent terminology, two different names for the same thing. The weight force is often indicated by a w. Again, it’s a vector, so again, it would be, the weight force would be the mass of the object times g, where this g is still the 9.8 newtons per kilogram. This is how we will deal with weight force and gravitational forces in this class, but I thought it relevant to bring up why the moon goes around the earth, how this is connected to falling objects, because it demonstrates the power of fundamental ideas.

Normal Force
Exercise:

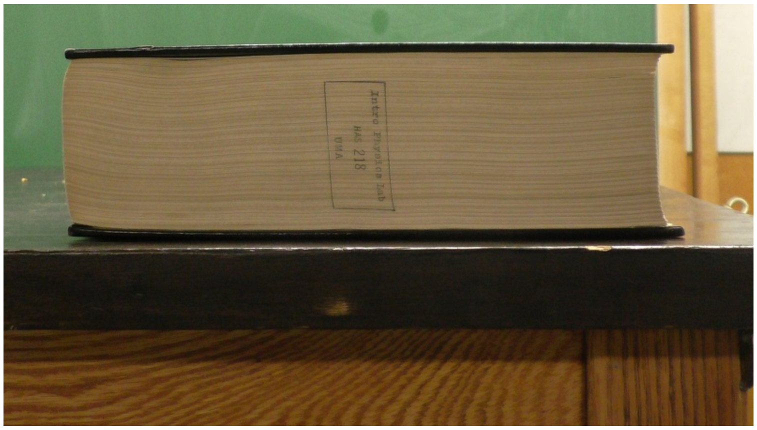
UMASS AMHERST Instructor's Notes

Problem:
Your Quiz will Cover

- Defining normal force in your own terms
- Identifying that normal force is a constraint force, i.e. has no formula

This section is also available as a video at https://www.youtube.com/watch?v=xNZYD3UhT48.

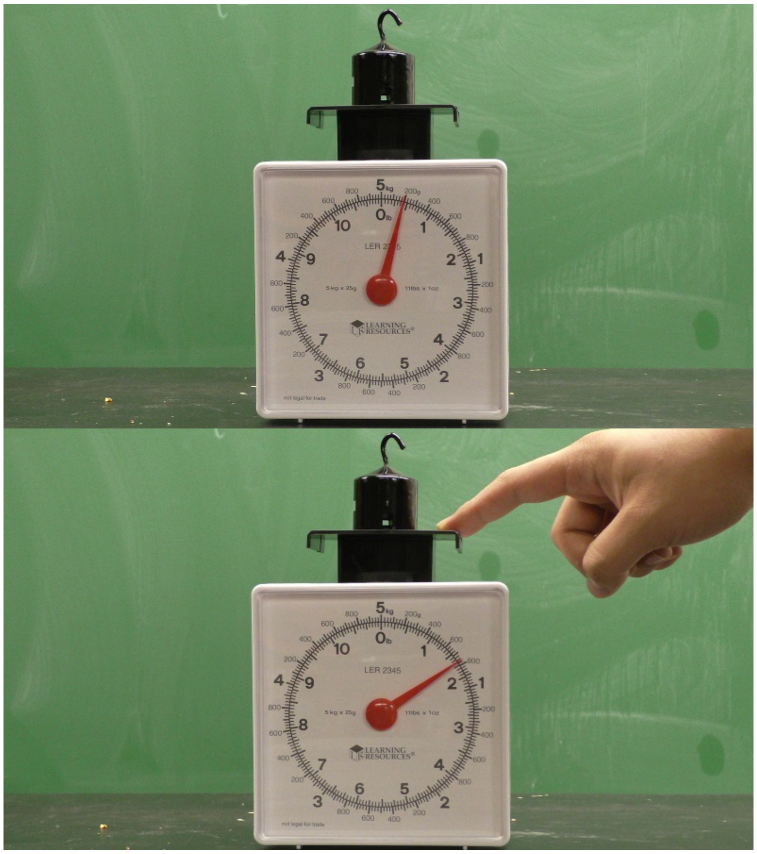
The goal of this section is to introduce you to some of the basic properties of the normal force. So, what is the normal force? The normal force is the force that keeps objects from passing through each other. A common example is that of a book sitting on a table. The force of gravity tries to pull the book down through the table; clearly, the book does not go through the table, and so we are forced to conclude that there must be some type of force from the table on the book pushing upwards to balance this. We call this the **normal force**. In general, a normal force can be thought of as any time one object pushes on another. In the example of the book on the table, the table is pushing on the book upward, keeping it from falling through.



A book sitting on top of a table. The normal force from the table on the book balances the force of gravity from the Earth on the book, preventing the book from falling through the table.

Another example with an active agent is a person pushing on a box. What is the force on the box? It’s a normal force from the hand on the box at the molecular level. The normal force arises from the electrons in one surface repelling the electrons in the other surface, but we overlook this microscopic level detail and just call the net effect the normal force.

It’s worth taking a few minutes to talk about the connections between the normal force and a scale. The question essentially is, what do scales actually read? Well, you might think that a scale just reads the amount of weight put on it. In the picture, a 500-gram weight is placed on a scale, and the scale reads 500 grams. However, that’s not all scales can measure. When the scale is pushed down on, you can see that the number goes up. So, what does this scale actually measure? It measures the amount of force being applied to the scale. In essence, this scale reads the normal force.



A 500g weight resting on top of a scale. The scale reads the force exerted on it, which is the force of the weight and the force exerted by the finger.

in summary, scales measure the force with which you press on them. They measure the force with which one object, my finger and this weight, push on another the platform of the scale. Scales measure the normal force. This is an important fact to remember as we will be using a variant of a scale known as a force plate in class.

So, let’s summarize the characteristics of the normal force. The normal force is a contact force. The two objects must be in contact for the normal force to be present. The normal force is also a constraint force. This means that there’s no formula for the magnitude of the normal force; it takes on whatever value is needed to keep Newton’s second law true. It’s also

important to remember that the normal force is always, well, normal. Normal means perpendicular in mathematic-ese, and the normal force is always perpendicular to the surface. Finally, it’s important to remember that scales measure the normal force.

Tension

Exercise:

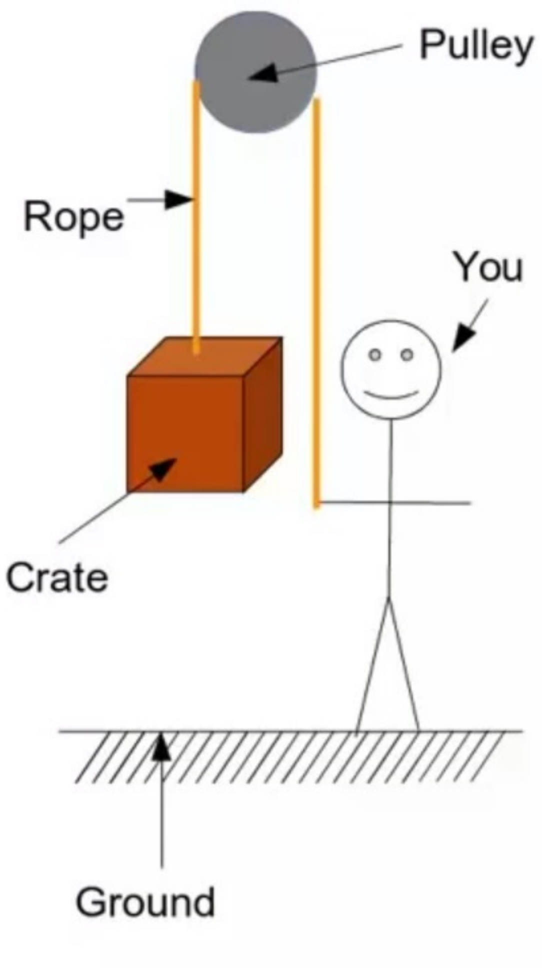
UMASS AMHERST Instructor's Notes

Problem:

This section has two parts. The first part is an example with a pulley to help you develop an understanding of tension from an everyday example, and the second part is the text from the OpenStax book.

The first example is also available as a video at https://www.youtube.com/watch?v=J_VSrAREPLY.

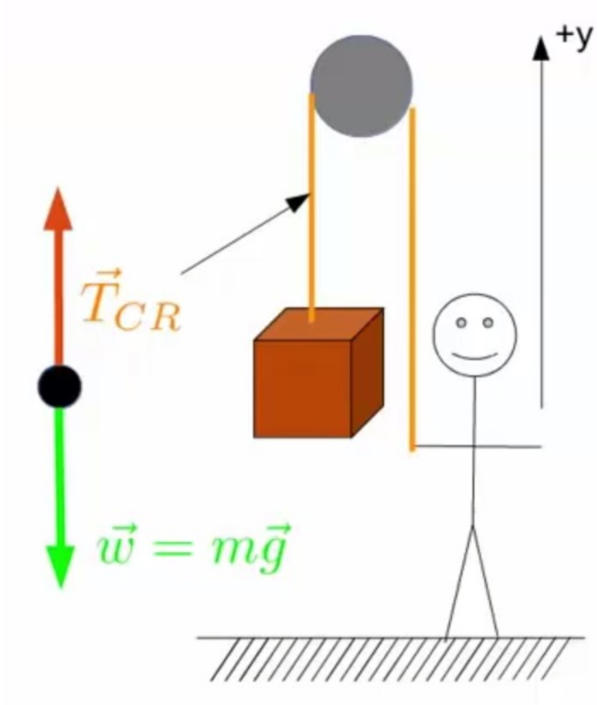
To explore this tension, let’s consider a crate suspended above the ground, held up by a rope.



This rope goes over a pulley to you at the other end of the rope. Now, it may be obvious to you the answer to the question “what holds up the stationary crate?”. Well, the rope does, but we want to explore this situation using our physics language.

Let’s consider this situation from a physics perspective. What do we know about the crate? Well, we know that the crate is stationary. The fact that the crate is stationary means that the velocity isn’t changing with respect to time. In other words, there is no net acceleration. By Newton’s second law, no net acceleration implies that there is zero net force, a fact which we can write mathematically like this.

When considering forces in physics, it’s often helpful to draw free body diagrams. Here’s one for this example:



We’ll model the crate as a black dot. What else do we know about this problem? Well, we’ve already determined that the net force has to be zero, which means that there must be some other force pointing in opposition to the force of the weight. By vector addition, if the weight points down and the net force is zero, then this other force must point up. This force is due to the tension within the rope, and so we call it the tension force, and it acts on the crate from the rope. Now let’s apply Newton’s second law to this situation. Remember, the crate is stationary, so there’s no acceleration. We’ll start by just stating Newton’s second law mathematically, the sum of the forces is equal to the mass times the acceleration. The forces in this case include the tension on the crate from the rope and the weight, which we’ve already decided to model as mg. We’ve already determined that the acceleration must be zero because the crate is stationary. Now, both the tension and the weight force are vectors. Since we’re adding them up, we need to break them into components.

In order to break them into components, we need to establish a coordinate system. We define the y-direction to be positive going up. With this convention, the tension on the crate from the rope is positive, and the weight is negative. Doing the algebra gives us the tension on the crate from the rope is equal to the mass of the crate multiplied by little G which, on

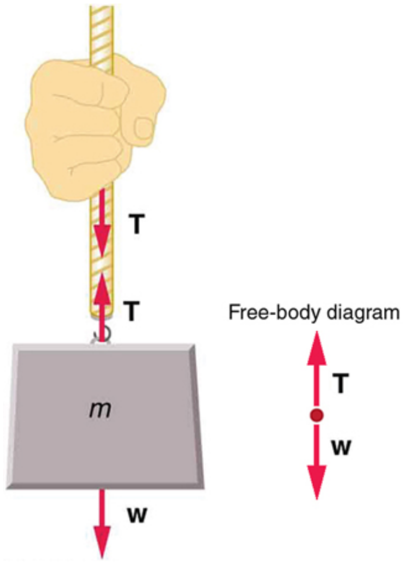
earth recall, is 9.8 meters per second squared. Would this equality between the tension and the rope and the weight of the box be true if the box were accelerating? No. Why not? Because this equality between the tension in the way only arose because we set the acceleration equal to 0 here in the second step. If that was not true then the tension would not be equal to the weight. What about you on the other end of the rope? How hard do you have to pull down? One property of ropes that we will make extensive use of in this class is that the tension in a rope is the same everywhere. What do I mean by this? Well, we solve for the tension over here where the crate is connected to the rope. By analyzing this part of the system, we can conclude that the tension on the crate from the rope was equal to the weight of the crate.

The fact that the tension must be the same everywhere in the rope means that the tension in the rope where it meets the hand is also equal to mg. Therefore, the rope is pulling up on my hand with the tension force equal to the weight of the crate, mg, which means that if I want everything to stay stationary I have to pull down with a force equal to the weight of the crate, mg, to keep everything in place. This should be in conjunction with your everyday experience, where to lift a box using any type of pulley, you've got to pull down with at least the weight of the box. The pulley makes it easier because you're pulling down using your weight to help lift the box as opposed lifting it.

Tension

A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word "tension" comes from a Latin word meaning "to stretch." Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: "You can't push a rope." The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in [link].



When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force $\mathbf{T}$, that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope).

This is an example of Newton's third law. The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton's second law. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus $\mathbf{F}_{\text{net}} = 0$. The only external forces acting on the mass are its weight $\mathbf{w}$ and the tension $\mathbf{T}$ supplied by the rope. Thus,

Equation:

$$F_{\text{net}} = T - w = 0,$$

where T and w are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

Equation:

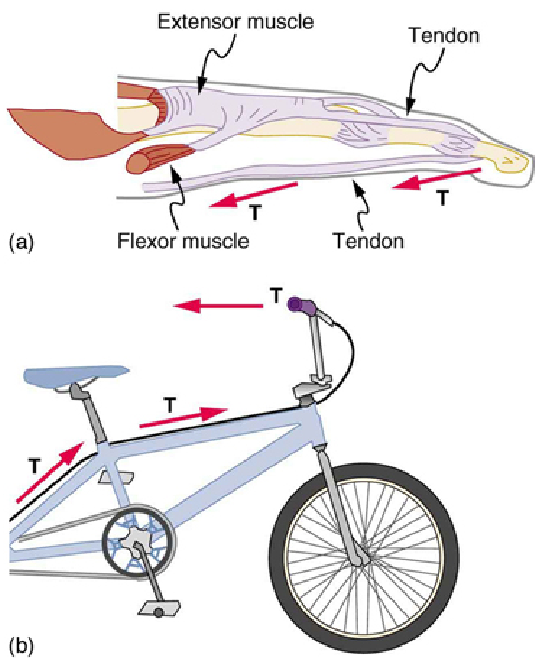
$$T = w = \text{mg}.$$

For a 5.00-kg mass, then (neglecting the mass of the rope) we see that
Equation:

$$T = \text{mg} = (5.00\ \text{kg})(9.80\ \text{m/s}^2) = 49.0\ \text{N}.$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in [link] (a) and (b).

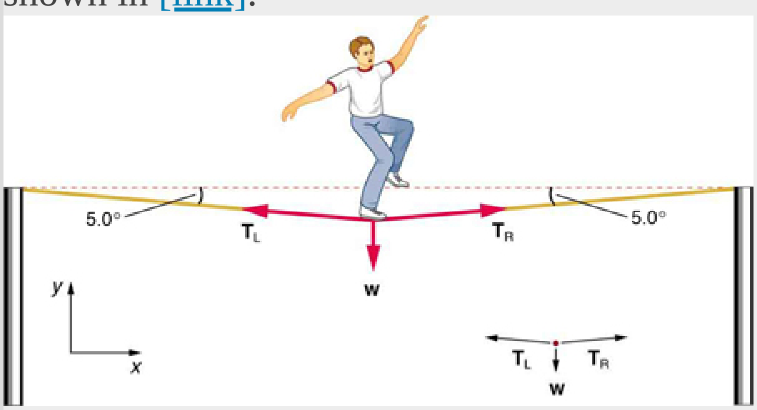


(a) Tendons in the finger carry force $\mathbf{T}$ from the

muscles to other parts of the finger, usually changing the force’s direction, but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension $\mathbf{T}$ from the handlebars to the brake mechanism. Again, the direction but not the magnitude of $\mathbf{T}$ is changed.

Example:
What Is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in [link].



The weight of a tightrope walker causes a wire to sag by 5.0 degrees. The system of interest here is the point in the wire at which the tightrope walker is standing.

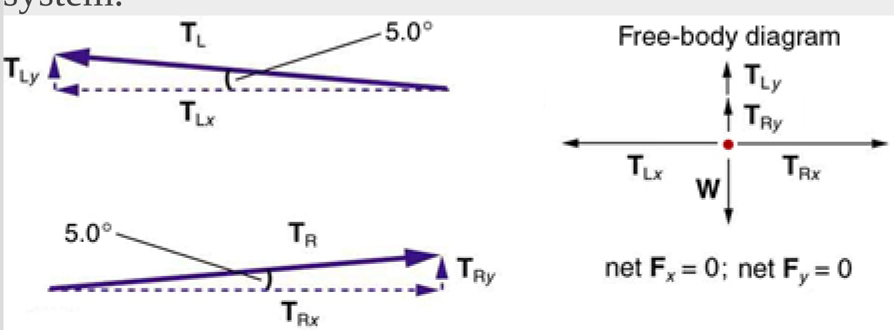
Strategy

As you can see in the figure, the wire is not perfectly horizontal (it cannot be!), but is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As usual, forces are vectors represented pictorially by arrows having the same directions as the forces and lengths proportional to their magnitudes. The system is the tightrope walker, and the only external forces acting on him are his weight $\mathbf{w}$ and the two tensions $\mathbf{T}_\mathrm{L}$ (left tension) and $\mathbf{T}_\mathrm{R}$ (right tension), as illustrated. It is reasonable to neglect the weight of the wire itself. The net external force is zero since the system is stationary. A little trigonometry can now be used to find the tensions. One conclusion is possible at the outset—we can see from part (b) of the figure that the magnitudes of the tensions T_L and T_R must be equal. This is because there is no horizontal acceleration in the rope, and the only forces acting to the left and right are T_L and T_R. Thus, the magnitude of those forces must be equal so that they cancel each other out.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and project the vectors onto its axes. In this case the best coordinate system has one axis horizontal and the other vertical. We call the horizontal the x-axis and the vertical the y-axis.

Solution

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to draw a new free-body diagram showing all of the horizontal and vertical components of each force acting on the system.



When the vectors are projected onto vertical and horizontal axes, their components along those axes must add to zero, since the tightrope walker is

stationary. The small angle results in T being much greater than w.

Consider the horizontal components of the forces (denoted with a subscript x):

Equation:

$$F_{\text{net}x} = T_{\text{L}x} - T_{\text{R}x}.$$

The net external horizontal force $F_{\text{net}x} = 0$, since the person is stationary. Thus,

Equation:

$$\begin{array}{lll} F_{\text{net}x} = 0 & = & T_{\text{L}x} - T_{\text{R}x} \\ T_{\text{L}x} & = & T_{\text{R}x}. \end{array}$$

Now, observe [link]. You can use trigonometry to determine the magnitude of T_{L} and T_{R}. Notice that:

Equation:

$$\begin{array}{lll} \cos\left(5.0^{\text{o}}\right) & = & \frac{T_{\text{L}x}}{T_{\text{L}}} \\ T_{\text{L}x} & = & T_{\text{L}} \cos\left(5.0^{\text{o}}\right) \\ \cos\left(5.0^{\text{o}}\right) & = & \frac{T_{\text{R}x}}{T_{\text{R}}} \\ T_{\text{R}x} & = & T_{\text{R}} \cos\left(5.0^{\text{o}}\right). \end{array}$$

Equating $T_{\text{L}x}$ and $T_{\text{R}x}$:

Equation:

$$T_{\text{L}} \cos\left(5.0^{\text{o}}\right) = T_{\text{R}} \cos\left(5.0^{\text{o}}\right).$$

Thus,

Equation:

$$T_{\text{L}} = T_{\text{R}} = T,$$

as predicted. Now, considering the vertical components (denoted by a subscript y), we can solve for T. Again, since the person is stationary, Newton’s second law implies that net $F_y = 0$. Thus, as illustrated in the free-body diagram in [link],

Equation:

$$F_{\text{net}y} = T_{\text{L}y} + T_{\text{R}y} - w = 0.$$

Observing [link], we can use trigonometry to determine the relationship between $T_{\text{L}y}$, $T_{\text{R}y}$, and T. As we determined from the analysis in the horizontal direction, $T_{\text{L}} = T_{\text{R}} = T$:

Equation:

$$\begin{array}{lll}
\text{sin}\ (5.0^{\text{o}}) & = & \frac{T_{\text{L}y}}{T_{\text{L}}} \\
T_{\text{L}y} = T_{\text{L}}\ \text{sin}\ (5.0^{\text{o}}) & = & T\ \text{sin}\ (5.0^{\text{o}}) \\
\text{sin}\ (5.0^{\text{o}}) & = & \frac{T_{\text{R}y}}{T_{\text{R}}} \\
T_{\text{R}y} = T_{\text{R}}\ \text{sin}\ (5.0^{\text{o}}) & = & T\ \text{sin}\ (5.0^{\text{o}}).
\end{array}$$

Now, we can substitute the values for $T_{\text{L}y}$ and $T_{\text{R}y}$, into the net force equation in the vertical direction:

Equation:

$$\begin{array}{lll}
F_{\text{net}y} & = & T_{\text{L}y} + T_{\text{R}y} - w = 0 \\
F_{\text{net}y} & = & T\ \text{sin}\ (5.0^{\text{o}}) + T\ \text{sin}\ (5.0^{\text{o}}) - w = 0 \\
2\ T\ \text{sin}\ (5.0^{\text{o}}) - w & = & 0 \\
2\ T\ \text{sin}\ (5.0^{\text{o}}) & = & w
\end{array}$$

and

Equation:

$$T = \frac{w}{2\ \text{sin}\ (5.0^{\text{o}})} = \frac{\text{mg}}{2\ \text{sin}\ (5.0^{\text{o}})},$$

so that

Equation:

$$T = \frac{(70.0\ \text{kg})(9.80\ \text{m/s}^2)}{2(0.0872)},$$

and the tension is
Equation:

$$T = 3900\ \text{N}.$$

Discussion
Note that the vertical tension in the wire acts as a normal force that supports the weight of the tightrope walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a small fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, and so most of the tension in the wire is not used to support the weight of the tightrope walker.

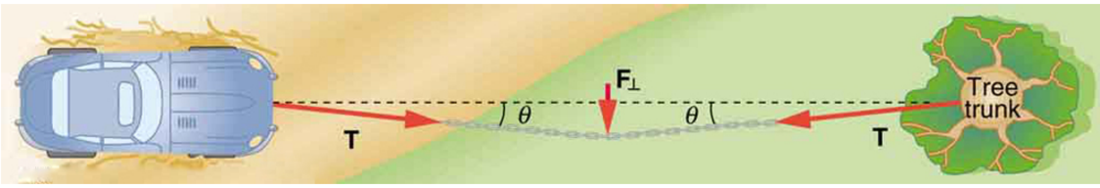
If we wish to *create* a very large tension, all we have to do is exert a force perpendicular to a flexible connector, as illustrated in [link]. As we saw in the last example, the weight of the tightrope walker acted as a force perpendicular to the rope. We saw that the tension in the roped related to the weight of the tightrope walker in the following way:
Equation:

$$T = \frac{w}{2\ \text{sin}\ (\theta)}.$$

We can extend this expression to describe the tension T created when a perpendicular force ($\mathbf{F}_\perp$) is exerted at the middle of a flexible connector:
Equation:

$$T = \frac{F_\perp}{2\ \text{sin}\ (\theta)}.$$

Note that θ is the angle between the horizontal and the bent connector. In this case, T becomes very large as θ approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite tension would result if it were horizontal (i.e., $\theta = 0$ and $\sin\theta = 0$). (See [link].)



We can create a very large tension in the chain by pushing on it perpendicular to its length, as shown. Suppose we wish to pull a car out of the mud when no tow truck is available. Each time the car moves forward, the chain is tightened to keep it as nearly straight as possible. The tension in the chain is given by $T = \frac{F_\perp}{2\sin(\theta)}$; since θ is small, T is very large. This situation is analogous to the tightrope walker shown in [link], except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where $\mathbf{F}_\perp$ is applied.

Unless an infinite tension is exerted, any flexible connector—such as the chain at the bottom of the picture—will sag under its own weight, giving a characteristic curve when the weight is evenly distributed along the length. Suspension bridges—such as the Golden Gate Bridge shown in this image—are essentially very heavy flexible connectors. The weight of the bridge is evenly distributed along the length of flexible connectors, usually cables, which take on the characteristic shape. (credit: Leaflet, Wikimedia Commons)

Glossary

inertial frame of reference
: a coordinate system that is not accelerating; all forces acting in an inertial frame of reference are real forces, as opposed to fictitious

forces that are observed due to an accelerating frame of reference

normal force
: the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

tension
: the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

Friction

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitude of static and kinetic friction.

Friction is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

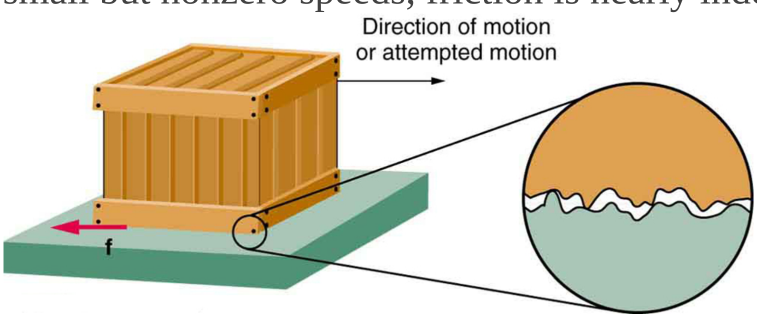
Note:
Friction
Friction is a force that opposes relative motion between systems in contact.

One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, **static friction** can act between them; the static friction is usually greater than the kinetic friction between the objects.

Note:
Kinetic Friction
If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

[link] is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.



Frictional forces, such as f, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of

the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the **magnitude of static friction $\mathbf{f}_{\mathrm{s}}$** is

Equation:

$$f_{\mathrm{s}} \leq \mu_{\mathrm{s}} N,$$

where μ_{s} is the coefficient of static friction and N is the magnitude of the normal force (the force perpendicular to the surface).

Note:
Magnitude of Static Friction
Magnitude of static friction f_{s} is

Equation:

$$f_{\mathrm{s}} \leq \mu_{\mathrm{s}} N,$$

where μ_{s} is the coefficient of static friction and N is the magnitude of the normal force.

The symbol $\leq$ means *less than or equal to*, implying that static friction can have a minimum and a maximum value of $\mu_{\mathrm{s}} N$. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds $f_{\mathrm{s(max)}}$, the object will move. Thus

Equation:

$$f_{\mathrm{s(max)}} = \mu_{\mathrm{s}} N.$$

Once an object is moving, the **magnitude of kinetic friction $\mathbf{f}_{\mathrm{k}}$** is given by

Equation:

$$f_{\mathrm{k}} = \mu_{\mathrm{k}} N,$$

where μ_{k} is the coefficient of kinetic friction. A system in which $f_{\mathrm{k}} = \mu_{\mathrm{k}} N$ is described as a system in which *friction behaves simply*.

Note:

Magnitude of Kinetic Friction

The magnitude of kinetic friction f_{k} is given by

Equation:

$$f_{\mathrm{k}} = \mu_{\mathrm{k}} N,$$

where μ_{k} is the coefficient of kinetic friction.

As seen in [link], the coefficients of kinetic friction are less than their static counterparts. That values of μ in [link] are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

System	**Static friction** μ_s	**Kinetic friction** μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7

System	Static friction μ_{s}	Kinetic friction μ_{k}
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.04	0.02

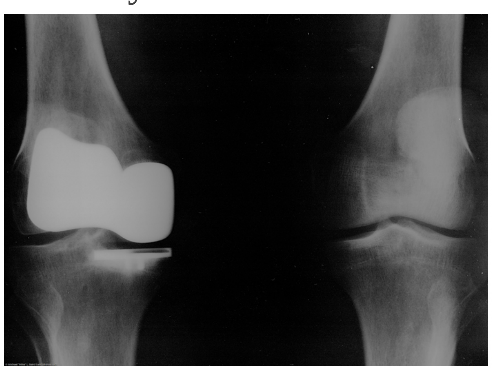
Coefficients of Static and Kinetic Friction

The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight, $W = mg = (100\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = 980\ \mathrm{N}$, perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than $f_{\mathrm{s(max)}} = \mu_{\mathrm{s}} N = (0.45)(980\ \mathrm{N}) = 440\ \mathrm{N}$ to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N ($f_{\mathrm{k}} = \mu_{\mathrm{k}} N = (0.30)(980\ \mathrm{N}) = 290\ \mathrm{N}$) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

Note:
Take-Home Experiment
Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table,

simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint ([link]). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.



Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op x rays of the right knee joint replacement. (credit: Mike Baird, Flickr)

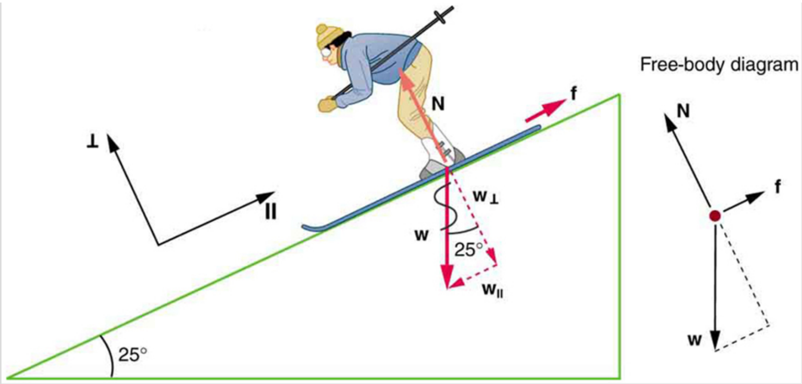
Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor's clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to mover freely over the skin.

Example:
Skiing Exercise

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

Strategy

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force N as $f_{\mathrm{k}} = \mu_{\mathrm{k}} N$; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See the skier and free-body diagram in [link].)



The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). **N** (the normal force) is perpendicular to the slope, and **f** (the friction) is parallel to the slope, but **w** (the skier's weight) has components along both axes, namely $\mathbf{w}_{\perp}$ and $\mathbf{W}_{//}$. **N** is equal in magnitude to $\mathbf{w}_{\perp}$, so there is no motion perpendicular to the slope. However, **f** is less than $\mathbf{W}_{//}$ in magnitude, so there is acceleration down the slope (along the *x*-axis).

That is,

Equation:

$$N = w_{\perp} = w\cos 25^{\mathrm{o}} = mg\cos 25^{\mathrm{o}}.$$

Substituting this into our expression for kinetic friction, we get

Equation:

$$f_{\mathrm{k}} = \mu_{\mathrm{k}}\mathrm{mg}\cos 25^{\mathrm{o}},$$

which can now be solved for the coefficient of kinetic friction μ_{k}.

Solution

Solving for μ_k gives

Equation:

$$\mu_\mathrm{k} = \frac{f_\mathrm{k}}{N} = \frac{f_\mathrm{k}}{w\cos 25^\mathrm{o}} = \frac{f_\mathrm{k}}{\mathrm{mg}\cos 25^\mathrm{o}.}$$

Substituting known values on the right-hand side of the equation,

Equation:

$$\mu_\mathrm{k} = \frac{45.0\ \mathrm{N}}{(62\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(0.906)} = 0.082.$$

Discussion

This result is a little smaller than the coefficient listed in [link] for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $f_\mathrm{k} = \mu_\mathrm{k}\mathrm{mg}\cos\theta$. All objects will slide down a slope with constant acceleration under these circumstances. Proof of this is left for this chapter's Problems and Exercises.

Note:

Take-Home Experiment

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in [link], the kinetic friction on a slope $f_\mathrm{k} = \mu_\mathrm{k}\mathrm{mg}\ \cos\theta$. The component of the weight down the slope is equal to $\mathrm{mg}\ \sin\theta$ (see the free-body diagram in [link]). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out:

Equation:

$$f_\mathrm{k} = \mathrm{Fg_x}$$

Equation:

$$\mu_{\mathrm{k}} \mathrm{mg} \cos \theta = \mathrm{mg} \sin \theta .$$

Solving for μ_{k}, we find that

Equation:

$$\mu_{\mathrm{k}} = \frac{\mathrm{mg} \sin \theta}{\mathrm{mg} \cos \theta} = \tan \theta .$$

Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find μ_{k}. Note that the coin will not start to slide at all until an angle greater than θ is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Discuss how this may affect the value for μ_{k} and its uncertainty.

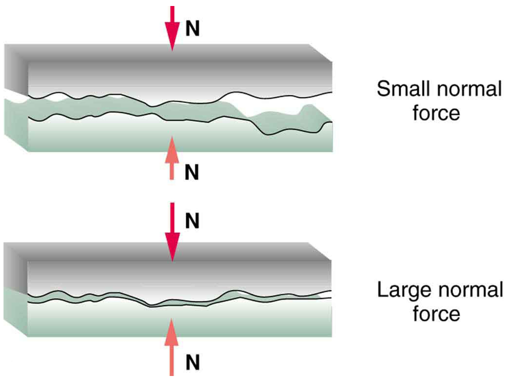
We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

Note:

Making Connections: Submicroscopic Explanations of Friction

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

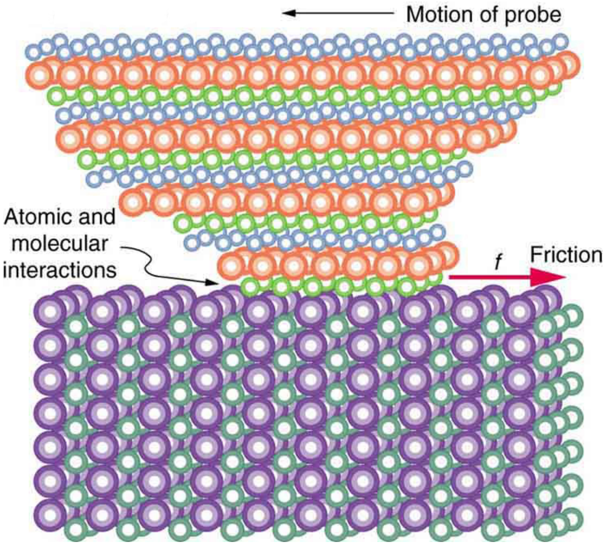
[link] illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.



Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction.

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate —essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur

between atoms and molecules on the surfaces. [link] shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of 10^{12}) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.



The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

Note:
PhET Explorations: Forces and Motion

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).

Forces and Motion

Section Summary

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force N pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction f_s between systems stationary relative to one another is given by
  Equation:
  $$f_s \leq \mu_s N,$$
  where μ_s is the coefficient of static friction, which depends on both of the materials.
- The kinetic friction force f_k between systems moving relative to one another is given by
  Equation:
  $$f_k = \mu_k N,$$

where μ_k is the coefficient of kinetic friction, which also depends on both materials.

Conceptual Questions

Exercise:

Problem:

Define normal force. What is its relationship to friction when friction behaves simply?

Exercise:

Problem:

The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.

Exercise:

Problem:

When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.

Exercise:

Problem:

When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular explaining how it is related to the fact that kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)

Problems & Exercises

Exercise:

Problem:

A physics major is cooking breakfast when he notices that the frictional force between his steel spatula and his Teflon frying pan is only 0.200 N. Knowing the coefficient of kinetic friction between the two materials, he quickly calculates the normal force. What is it?

Solution:

5.00 N

Exercise:

Problem:

(a) When rebuilding her car's engine, a physics major must exert 300 N of force to insert a dry steel piston into a steel cylinder. What is the magnitude of the normal force between the piston and cylinder? (b) What is the magnitude of the force would she have to exert if the steel parts were oiled?

Exercise:

Problem:

(a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise it is possible to exert forces to the joints that are easily ten times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.

Exercise:

Problem:

Suppose you have a 120-kg wooden crate resting on a wood floor. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will the magnitude of its acceleration then be?

Solution:

(a) 588 N

(b) $1.96\ \text{m/s}^2$

Exercise:

Problem:

(a) If half of the weight of a small 1.00×10^3 kg utility truck is supported by its two drive wheels, what is the magnitude of the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.

Exercise:

Problem:

A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the magnitude of the acceleration starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) What is the magnitude of the acceleration once the sled starts to move? (c) For both situations, calculate the magnitude of the force in the coupling between the dogs and the sled.

Solution:

(a) 3.29 m/s^2

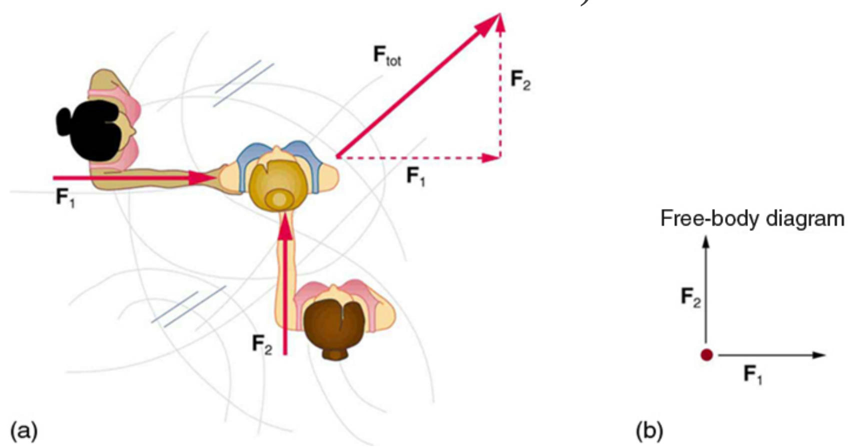
(b) 3.52 m/s^2

(c) 980 N; 945 N

Exercise:

Problem:

Consider the 65.0-kg ice skater being pushed by two others shown in [link]. (a) Find the direction and magnitude of $\mathbf{F}_{\text{tot}}$, the total force exerted on her by the others, given that the magnitudes F_1 and F_2 are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of $\mathbf{F}_{\text{tot}}$? (c) What is her acceleration assuming she is already moving in the direction of $\mathbf{F}_{\text{tot}}$? (Remember that friction always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)



Exercise:

Problem:

Show that the acceleration of any object down a frictionless incline that makes an angle θ with the horizontal is $a = g \sin \theta$. (Note that this acceleration is independent of mass.)

Exercise:

Problem:

Show that the acceleration of any object down an incline where friction behaves simply (that is, where $f_k = \mu_k N$) is $a = g(\sin\theta - \mu_k \cos\theta)$. Note that the acceleration is independent of mass and reduces to the expression found in the previous problem when friction becomes negligibly small $(\mu_k = 0)$.

Exercise:

Problem:

Calculate the deceleration of a snow boarder going up a 5.0^{o}, slope assuming the coefficient of friction for waxed wood on wet snow. The result of [link] may be useful, but be careful to consider the fact that the snow boarder is going uphill. Explicitly show how you follow the steps in Problem-Solving Strategies.

Solution:

$1.83\ \text{m/s}^2$

Exercise:

Problem:

(a) Calculate the acceleration of a skier heading down a 10.0^{o} slope, assuming the coefficient of friction for waxed wood on wet snow. (b) Find the angle of the slope down which this skier could coast at a constant velocity. You can neglect air resistance in both parts, and you will find the result of [link] to be useful. Explicitly show how you follow the steps in the Problem-Solving Strategies.

Exercise:

Problem:

If an object is to rest on an incline without slipping, then friction must equal the component of the weight of the object parallel to the incline. This requires greater and greater friction for steeper slopes. Show that the maximum angle of an incline above the horizontal for which an object will not slide down is $\theta = \tan^{-1} \mu_s$. You may use the result of the previous problem. Assume that $a = 0$ and that static friction has reached its maximum value.

Exercise:

Problem:

Calculate the maximum deceleration of a car that is heading down a 6^o slope (one that makes an angle of 6^o with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the deceleration. (Ignore rolling.) Calculate for a car: (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s = 0.100$, the same as for shoes on ice.

Exercise:

Problem:

Calculate the maximum acceleration of a car that is heading up a 4^o slope (one that makes an angle of 4^o with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s = 0.100$, the same as for shoes on ice.

Solution:

(a) $4.20\ \mathrm{m/s}^2$

(b) 2.74 m/s^2

(c) –0.195 m/s^2

Exercise:

Problem: Repeat [link] for a car with four-wheel drive.

Exercise:

Problem:

A freight train consists of two 8.00×10^5-kg engines and 45 cars with average masses of 5.50×10^5 kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of $5.00 \times 10^{-2} \text{ m/s}^2$ if the force of friction is 7.50×10^5 N, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the magnitude of the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

Solution:

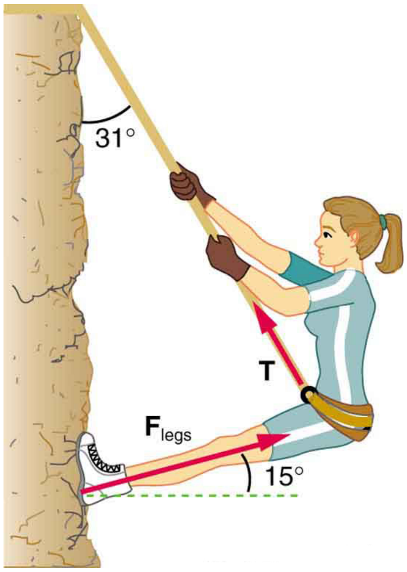
(a) 1.03×10^6 N

(b) 3.48×10^5 N

Exercise:

Problem:

Consider the 52.0-kg mountain climber in [link]. (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?



Part of the climber's weight is supported by her rope and part by friction between her feet and the rock face.

Exercise:

Problem:

A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown in [link](a). (a) Calculate the minimum force F he must exert to get the block moving. (b) What is the magnitude of its acceleration once it starts to move, if that force is maintained?

Solution:

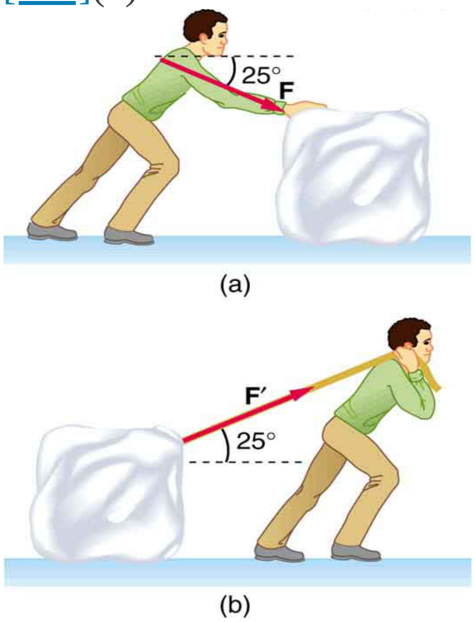
(a) 51.0 N

(b) $0.720\ \text{m/s}^2$

Exercise:

Problem:

Repeat [link] with the contestant pulling the block of ice with a rope over his shoulder at the same angle above the horizontal as shown in [link](b).



Which method of sliding a block of ice requires less force—(a) pushing or (b) pulling at the same angle above the horizontal?

Glossary

friction
: a force that opposes relative motion or attempts at motion between systems in contact

kinetic friction

a force that opposes the motion of two systems that are in contact and moving relative to one another

static friction
a force that opposes the motion of two systems that are in contact and are not moving relative to one another

magnitude of static friction
$f_s \leq \mu_s N$, where μ_s is the coefficient of static friction and N is the magnitude of the normal force

magnitude of kinetic friction
$f_k = \mu_k N$, where μ_k is the coefficient of kinetic friction

Elasticity: Stress and Strain

- State Hooke’s law.
- Explain Hooke’s law using graphical representation between deformation and applied force.
- Discuss the three types of deformations such as changes in length, sideways shear and changes in volume.
- Describe with examples the young’s modulus, shear modulus and bulk modulus.
- Determine the change in length given mass, length and radius.

We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object’s shape. If a bulldozer pushes a car into a wall, the car will not move but it will noticeably change shape. A change in shape due to the application of a force is a **deformation**. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force—that is, for small deformations, Hooke’s law is obeyed. In equation form, **Hooke’s law** is given by

Equation:

$$F = k\Delta L,$$

where ΔL is the amount of deformation (the change in length, for example) produced by the force F, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force. Note that this force is a function of the deformation ΔL —it is not constant as a kinetic friction force is. Rearranging this to

Equation:

$$\Delta L = \frac{F}{k}$$

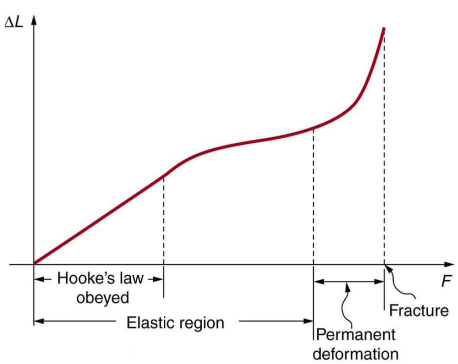
makes it clear that the deformation is proportional to the applied force. [link] shows the Hooke's law relationship between the extension ΔL of a spring or of a human bone. For metals or springs, the straight line region in which Hooke's law pertains is much larger. Bones are brittle and the elastic region is small and the fracture abrupt. Eventually a large enough stress to the material will cause it to break or fracture. **Tensile strength** is the breaking stress that will cause permanent deformation or fracture of a material.

Note:
Hooke's Law
Equation:

$$F = k\Delta L,$$

where ΔL is the amount of deformation (the change in length, for example) produced by the force F, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force.
Equation:

$$\Delta L = \frac{F}{k}$$

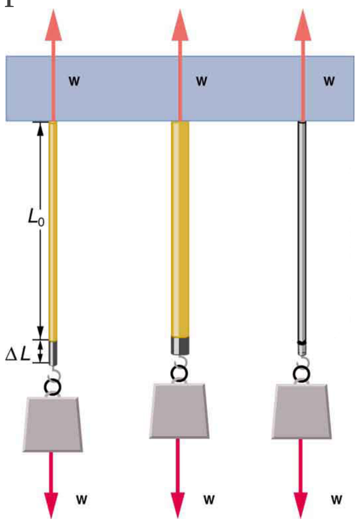


A graph of

deformation ΔL versus applied force F. The straight segment is the linear region where Hooke's law is obeyed. The slope of the straight region is $\frac{1}{k}$. For larger forces, the graph is curved but the deformation is still elastic— ΔL will return to zero if the force is removed. Still greater forces permanently deform the object until it finally fractures. The shape of the curve near fracture depends on several factors, including how the force F is applied. Note that in this graph the slope increases just before fracture, indicating that a small increase in F is producing a large increase in L near the fracture.

The proportionality constant k depends upon a number of factors for the material. For example, a guitar string made of nylon stretches when it is tightened, and the elongation ΔL is proportional to the force applied (at least for small deformations). Thicker nylon strings and ones made of steel

stretch less for the same applied force, implying they have a larger k (see [link]). Finally, all three strings return to their normal lengths when the force is removed, provided the deformation is small. Most materials will behave in this manner if the deformation is less than about 0.1% or about 1 part in 10^3.



The same force, in this case a weight (w), applied to three different guitar strings of identical length produces the three different deformations shown as shaded segments. The string on the left is thin nylon, the one in the middle is thicker nylon, and the one on the right is steel.

Note:
Stretch Yourself a Little
How would you go about measuring the proportionality constant k of a rubber band? If a rubber band stretched 3 cm when a 100-g mass was attached to it, then how much would it stretch if two similar rubber bands were attached to the same mass—even if put together in parallel or alternatively if tied together in series?

Section Summary

- Hooke's law is given by
  Equation:
  $$F = k\Delta L,$$
  where ΔL is the amount of deformation (the change in length), F is the applied force, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force. The relationship between the deformation and the applied force can also be written as
  Equation:
  $$\Delta L = \frac{1}{Y}\frac{F}{A}L_0,$$
  where Y is *Young's modulus*, which depends on the substance, A is the cross-sectional area, and L_0 is the original length.
- The ratio of force to area, $\frac{F}{A}$, is defined as *stress*, measured in N/m^2.
- The ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as *strain* (a unitless quantity). In other words,

Equation:

$$\text{stress} = Y \times \text{strain}.$$

- The expression for shear deformation is
  Equation:

  $$\Delta x = \frac{1}{S}\frac{F}{A}L_0,$$

  where S is the shear modulus and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A.
- The relationship of the change in volume to other physical quantities is given by
  Equation:

  $$\Delta V = \frac{1}{B}\frac{F}{A}V_0,$$

  where B is the bulk modulus, V_0 is the original volume, and $\frac{F}{A}$ is the force per unit area applied uniformly inward on all surfaces.

Conceptual Questions

Exercise:

Problem:

The elastic properties of the arteries are essential for blood flow. Explain the importance of this in terms of the characteristics of the flow of blood (pulsating or continuous).

Exercise:

Problem:

What are you feeling when you feel your pulse? Measure your pulse rate for 10 s and for 1 min. Is there a factor of 6 difference?

Exercise:

Problem:

Examine different types of shoes, including sports shoes and thongs. In terms of physics, why are the bottom surfaces designed as they are? What differences will dry and wet conditions make for these surfaces?

Exercise:

Problem:

Would you expect your height to be different depending upon the time of day? Why or why not?

Exercise:

Problem:

Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

Exercise:

Problem:

Explain why pregnant women often suffer from back strain late in their pregnancy.

Exercise:

Problem:

An old carpenter’s trick to keep nails from bending when they are pounded into hard materials is to grip the center of the nail firmly with pliers. Why does this help?

Exercise:

Problem:

When a glass bottle full of vinegar warms up, both the vinegar and the glass expand, but vinegar expands significantly more with temperature than glass. The bottle will break if it was filled to its tightly capped lid. Explain why, and also explain how a pocket of air above the vinegar would prevent the break. (This is the function of the air above liquids in glass containers.)

Problems & Exercises

Exercise:

Problem:

During a circus act, one performer swings upside down hanging from a trapeze holding another, also upside-down, performer by the legs. If the upward force on the lower performer is three times her weight, how much do the bones (the femurs) in her upper legs stretch? You may assume each is equivalent to a uniform rod 35.0 cm long and 1.80 cm in radius. Her mass is 60.0 kg.

Solution:
Equation:

$$1.90 \times 10^{-3}\ \text{cm}$$

Exercise:

Problem:

During a wrestling match, a 150 kg wrestler briefly stands on one hand during a maneuver designed to perplex his already moribund adversary. By how much does the upper arm bone shorten in length? The bone can be represented by a uniform rod 38.0 cm in length and 2.10 cm in radius.

Exercise:

Problem:

(a) The “lead” in pencils is a graphite composition with a Young’s modulus of about $1 \times 10^9\ \mathrm{N/m^2}$. Calculate the change in length of the lead in an automatic pencil if you tap it straight into the pencil with a force of 4.0 N. The lead is 0.50 mm in diameter and 60 mm long. (b) Is the answer reasonable? That is, does it seem to be consistent with what you have observed when using pencils?

Solution:

(a)1 mm
(b) This does seem reasonable, since the lead does seem to shrink a little when you push on it.

Exercise:

Problem:

TV broadcast antennas are the tallest artificial structures on Earth. In 1987, a 72.0-kg physicist placed himself and 400 kg of equipment at the top of one 610-m high antenna to perform gravity experiments. By how much was the antenna compressed, if we consider it to be equivalent to a steel cylinder 0.150 m in radius?

Exercise:

Problem:

(a) By how much does a 65.0-kg mountain climber stretch her 0.800-cm diameter nylon rope when she hangs 35.0 m below a rock outcropping? (b) Does the answer seem to be consistent with what you have observed for nylon ropes? Would it make sense if the rope were actually a bungee cord?

Solution:

(a)9 cm
(b)This seems reasonable for nylon climbing rope, since it is not

supposed to stretch that much.

Exercise:

Problem:

A 20.0-m tall hollow aluminum flagpole is equivalent in stiffness to a solid cylinder 4.00 cm in diameter. A strong wind bends the pole much as a horizontal force of 900 N exerted at the top would. How far to the side does the top of the pole flex?

Exercise:

Problem:

As an oil well is drilled, each new section of drill pipe supports its own weight and that of the pipe and drill bit beneath it. Calculate the stretch in a new 6.00 m length of steel pipe that supports 3.00 km of pipe having a mass of 20.0 kg/m and a 100-kg drill bit. The pipe is equivalent in stiffness to a solid cylinder 5.00 cm in diameter.

Solution:

8.59 mm

Exercise:

Problem:

Calculate the force a piano tuner applies to stretch a steel piano wire 8.00 mm, if the wire is originally 0.850 mm in diameter and 1.35 m long.

Exercise:

Problem:

A vertebra is subjected to a shearing force of 500 N. Find the shear deformation, taking the vertebra to be a cylinder 3.00 cm high and 4.00 cm in diameter.

Solution:

Equation:

$$1.49 \times 10^{-7}\ \mathrm{m}$$

Exercise:

Problem:

A disk between vertebrae in the spine is subjected to a shearing force of 600 N. Find its shear deformation, taking it to have the shear modulus of $1 \times 10^{9}\ \mathrm{N/m^2}$. The disk is equivalent to a solid cylinder 0.700 cm high and 4.00 cm in diameter.

Exercise:

Problem:

When using a pencil eraser, you exert a vertical force of 6.00 N at a distance of 2.00 cm from the hardwood-eraser joint. The pencil is 6.00 mm in diameter and is held at an angle of 20.0^{o} to the horizontal. (a) By how much does the wood flex perpendicular to its length? (b) How much is it compressed lengthwise?

Solution:

(a) $3.99 \times 10^{-7}\ \mathrm{m}$

(b) $9.67 \times 10^{-8}\ \mathrm{m}$

Exercise:

Problem:

To consider the effect of wires hung on poles, we take data from [link], in which tensions in wires supporting a traffic light were calculated. The left wire made an angle 30.0^{o} below the horizontal with the top of its pole and carried a tension of 108 N. The 12.0 m tall hollow aluminum pole is equivalent in stiffness to a 4.50 cm diameter solid cylinder. (a) How far is it bent to the side? (b) By how much is it compressed?

Exercise:

Problem:

A farmer making grape juice fills a glass bottle to the brim and caps it tightly. The juice expands more than the glass when it warms up, in such a way that the volume increases by 0.2% (that is, $\Delta V/V_0 = 2 \times 10^{-3}$) relative to the space available. Calculate the magnitude of the normal force exerted by the juice per square centimeter if its bulk modulus is $1.8 \times 10^9\ \text{N/m}^2$, assuming the bottle does not break. In view of your answer, do you think the bottle will survive?

Solution:

$4 \times 10^6\ \text{N/m}^2$. This is about 36 atm, greater than a typical jar can withstand.

Exercise:

Problem:

(a) When water freezes, its volume increases by 9.05% (that is, $\Delta V/V_0 = 9.05 \times 10^{-2}$). What force per unit area is water capable of exerting on a container when it freezes? (It is acceptable to use the bulk modulus of water in this problem.) (b) Is it surprising that such forces can fracture engine blocks, boulders, and the like?

Exercise:

Problem:

This problem returns to the tightrope walker studied in [link], who created a tension of $3.94 \times 10^3\ \text{N}$ in a wire making an angle 5.0° below the horizontal with each supporting pole. Calculate how much this tension stretches the steel wire if it was originally 15 m long and 0.50 cm in diameter.

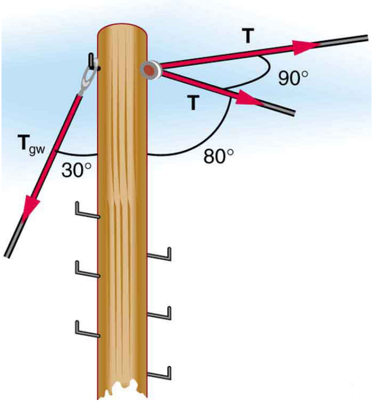
Solution:

1.4 cm

Exercise:

Problem:

The pole in [link] is at a 90.0° bend in a power line and is therefore subjected to more shear force than poles in straight parts of the line. The tension in each line is 4.00×10^4 N, at the angles shown. The pole is 15.0 m tall, has an 18.0 cm diameter, and can be considered to have half the stiffness of hardwood. (a) Calculate the compression of the pole. (b) Find how much it bends and in what direction. (c) Find the tension in a guy wire used to keep the pole straight if it is attached to the top of the pole at an angle of 30.0° with the vertical. (Clearly, the guy wire must be in the opposite direction of the bend.)



This telephone pole is at a 90° bend in a power line. A guy wire is attached to the top of the pole at an angle of 30° with the vertical.

Glossary

deformation
: change in shape due to the application of force

Hooke's law
: proportional relationship between the force F on a material and the deformation ΔL it causes, $F = k\Delta L$

tensile strength
: the breaking stress that will cause permanent deformation or fraction of a material

stress
: ratio of force to area

strain
: ratio of change in length to original length

shear deformation
: deformation perpendicular to the original length of an object

Drag Forces

- Express mathematically the drag force.
- Discuss the applications of drag force.
- Define terminal velocity.
- Determine the terminal velocity given mass.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This section has been included for completeness, and the content here may only be briefly touched upon in class.

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion. Like friction, the **drag force** always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. For most large objects such as bicyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force F_{D} is found to be proportional to the square of the speed of the object. We can write this relationship mathematically as $F_{\text{D}} \propto v^2$. When taking into account other factors, this relationship becomes

Equation:

$$F_{\text{D}} = \frac{1}{2}\text{C}\rho A v^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid. (Recall that density is mass per unit volume.) This equation can also be written in a more generalized fashion as $F_{\text{D}} = bv^2$, where b is a constant equivalent to $0.5C\rho A$. We have set the exponent for these equations as 2 because, when an object is moving at high velocity through air, the magnitude of the drag force is proportional to the square of the speed. As we shall see in a few pages on fluid dynamics, for small particles moving at low speeds in a fluid, the exponent is equal to 1.

Note:
Drag Force
Drag force F_{D} is found to be proportional to the square of the speed of the object. Mathematically
Equation:

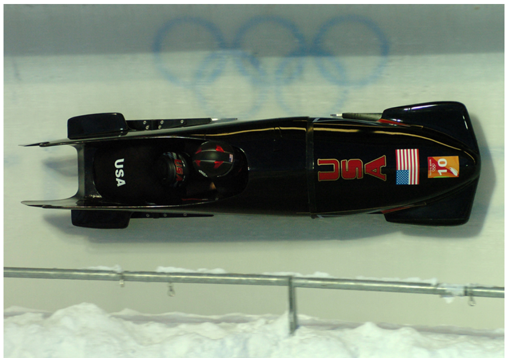
$$F_{\text{D}} \propto v^2$$

Equation:

$$F_{\text{D}} = \frac{1}{2} C\rho A v^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid.

Athletes as well as car designers seek to reduce the drag force to lower their race times. (See [link]). “Aerodynamic” shaping of an automobile can reduce the drag force and so increase a car’s gas mileage.



From racing cars to bobsled racers, aerodynamic shaping is crucial to achieving top speeds. Bobsleds are designed for speed. They are shaped like a bullet with tapered fins. (credit: U.S. Army, via Wikimedia Commons)

The value of the drag coefficient, C , is determined empirically, usually with the use of a wind tunnel. (See [link]).

NASA researchers

test a model plane in a wind tunnel. (credit: NASA/Ames)

The drag coefficient can depend upon velocity, but we will assume that it is a constant here. [link] lists some typical drag coefficients for a variety of objects. Notice that the drag coefficient is a dimensionless quantity. At highway speeds, over 50% of the power of a car is used to overcome air drag. The most fuel-efficient cruising speed is about 70–80 km/h (about 45–50 mi/h). For this reason, during the 1970s oil crisis in the United States, maximum speeds on highways were set at about 90 km/h (55 mi/h).

Object	***C***
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64

Object	***C***
Skydiver (feet first)	0.70
Bicycle	0.90
Skydiver (horizontal)	1.0
Circular flat plate	1.12

Drag Coefficient Values Typical values of drag coefficient C.

Substantial research is under way in the sporting world to minimize drag. The dimples on golf balls are being redesigned as are the clothes that athletes wear. Bicycle racers and some swimmers and runners wear full bodysuits. Australian Cathy Freeman wore a full body suit in the 2000 Sydney Olympics, and won the gold medal for the 400 m race. Many swimmers in the 2008 Beijing Olympics wore (Speedo) body suits; it might have made a difference in breaking many world records (See [link]). Most elite swimmers (and cyclists) shave their body hair. Such innovations can have the effect of slicing away milliseconds in a race, sometimes making the difference between a gold and a silver medal. One consequence is that careful and precise guidelines must be continuously developed to maintain the integrity of the sport.

Body suits, such as this LZR Racer Suit, have

been credited with many world records after their release in 2008. Smoother "skin" and more compression forces on a swimmer's body provide at least 10% less drag. (credit: NASA/Kathy Barnstorff)

Some interesting situations connected to Newton's second law occur when considering the effects of drag forces upon a moving object. For instance, consider a skydiver falling through air under the influence of gravity. The two forces acting on him are the force of gravity and the drag force (ignoring the buoyant force). The downward force of gravity remains constant regardless of the velocity at which the person is moving. However, as the person's velocity increases, the magnitude of the drag force increases until the magnitude of the drag force is equal to the gravitational force, thus producing a net force of zero. A zero net force means that there is no acceleration, as given by Newton's second law. At this point, the person's velocity remains constant and we say that the person has reached his *terminal velocity* (v_{t}). Since F_{D} is proportional to the speed, a heavier skydiver must go faster for F_{D} to equal his weight. Let's see how this works out more quantitatively.

At the terminal velocity,

Equation:

$$F_{\text{net}} = \text{mg} - F_{\text{D}} = \text{ma} = 0.$$

Thus,

Equation:

$$\text{mg} = F_{\text{D}}.$$

Using the equation for drag force, we have
Equation:

$$\mathrm{mg} = \frac{1}{2}\rho C A v^2.$$

Solving for the velocity, we obtain
Equation:

$$v = \sqrt{\frac{2\mathrm{mg}}{\rho \mathrm{CA}}}.$$

Assume the density of air is $\rho = 1.21\ \mathrm{kg/m^3}$. A 75-kg skydiver descending head first will have an area approximately $A = 0.18\ \mathrm{m}^2$ and a drag coefficient of approximately $C = 0.70$. We find that
Equation:

$$\begin{aligned} v &= \sqrt{\frac{2(75\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{(1.21\ \mathrm{kg/m^3})(0.70)(0.18\ \mathrm{m^2})}} \\ &= 98\ \mathrm{m/s} \\ &= 350\ \mathrm{km/h}. \end{aligned}$$

This means a skydiver with a mass of 75 kg achieves a maximum terminal velocity of about 350 km/h while traveling in a pike (head first) position, minimizing the area and his drag. In a spread-eagle position, that terminal velocity may decrease to about 200 km/h as the area increases. This terminal velocity becomes much smaller after the parachute opens.

Note:
Take-Home Experiment
This interesting activity examines the effect of weight upon terminal velocity. Gather together some nested coffee filters. Leaving them in their

original shape, measure the time it takes for one, two, three, four, and five nested filters to fall to the floor from the same height (roughly 2 m). (Note that, due to the way the filters are nested, drag is constant and only mass varies.) They obtain terminal velocity quite quickly, so find this velocity as a function of mass. Plot the terminal velocity v versus mass. Also plot v^2 versus mass. Which of these relationships is more linear? What can you conclude from these graphs?

Example:
A Terminal Velocity
Find the terminal velocity of an 85-kg skydiver falling in a spread-eagle position.
Strategy
At terminal velocity, $F_{\text{net}} = 0$. Thus the drag force on the skydiver must equal the force of gravity (the person's weight). Using the equation of drag force, we find $\text{mg} = \frac{1}{2}\rho CAv^2$.
Thus the terminal velocity v_{t} can be written as
Equation:

$$v_{\text{t}} = \sqrt{\frac{2\text{mg}}{\rho CA}}.$$

Solution
All quantities are known except the person's projected area. This is an adult (82 kg) falling spread eagle. We can estimate the frontal area as
Equation:

$$A = (2\text{ m})(0.35\text{ m}) = 0.70\text{ m}^2.$$

Using our equation for v_{t}, we find that
Equation:

$$\begin{aligned} v_t &= \sqrt{\frac{2(85\ \text{kg})(9.80\ \text{m/s}^2)}{(1.21\ \text{kg/m}^3)(1.0)(0.70\ \text{m}^2)}} \\ &= 44\ \text{m/s}. \end{aligned}$$

Discussion

This result is consistent with the value for v_t mentioned earlier. The 75-kg skydiver going feet first had a $v = 98\ \text{m/s}$. He weighed less but had a smaller frontal area and so a smaller drag due to the air.

The size of the object that is falling through air presents another interesting application of air drag. If you fall from a 5-m high branch of a tree, you will likely get hurt—possibly fracturing a bone. However, a small squirrel does this all the time, without getting hurt. You don't reach a terminal velocity in such a short distance, but the squirrel does.

The following interesting quote on animal size and terminal velocity is from a 1928 essay by a British biologist, J.B.S. Haldane, titled "On Being the Right Size."

To the mouse and any smaller animal, [gravity] presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, and a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.

The above quadratic dependence of air drag upon velocity does not hold if the object is very small, is going very slow, or is in a denser medium than air. Then we find that the drag force is proportional just to the velocity. This relationship is given by **Stokes' law**, which states that

Equation:

$$F_s = 6\pi r\eta v,$$

where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity.

Note:
Stokes' Law
Equation:

$$F_{\text{s}} = 6\pi r\eta v,$$

where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity.

Good examples of this law are provided by microorganisms, pollen, and dust particles. Because each of these objects is so small, we find that many of these objects travel unaided only at a constant (terminal) velocity. Terminal velocities for bacteria (size about $1\ \mu\text{m}$) can be about $2\ \mu\text{m/s}$. To move at a greater speed, many bacteria swim using flagella (organelles shaped like little tails) that are powered by little motors embedded in the cell. Sediment in a lake can move at a greater terminal velocity (about $5\ \mu\text{m/s}$), so it can take days to reach the bottom of the lake after being deposited on the surface.

If we compare animals living on land with those in water, you can see how drag has influenced evolution. Fishes, dolphins, and even massive whales are streamlined in shape to reduce drag forces. Birds are streamlined and migratory species that fly large distances often have particular features such as long necks. Flocks of birds fly in the shape of a spear head as the flock forms a streamlined pattern (see [link]). In humans, one important example of streamlining is the shape of sperm, which need to be efficient in their use of energy.

Geese fly in a V formation during their long migratory travels. This shape reduces drag and energy consumption for individual birds, and also allows them a better way to communicate. (credit: Julo, Wikimedia Commons)

Note:
Galileo’s Experiment
Galileo is said to have dropped two objects of different masses from the Tower of Pisa. He measured how long it took each to reach the ground. Since stopwatches weren’t readily available, how do you think he measured their fall time? If the objects were the same size, but with different masses, what do you think he should have observed? Would this result be different if done on the Moon?

Note:

PhET Explorations: Masses & Springs
A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energy for each spring.

Masses & Springs

Section Summary

- Drag forces acting on an object moving in a fluid oppose the motion. For larger objects (such as a baseball) moving at a velocity v in air, the drag force is given by
  Equation:
  $$F_{\mathrm{D}} = \frac{1}{2}C\rho A v^2,$$
  where C is the drag coefficient (typical values are given in [link]), A is the area of the object facing the fluid, and ρ is the fluid density.
- For small objects (such as a bacterium) moving in a denser medium (such as water), the drag force is given by Stokes' law,
  Equation:
  $$F_{\mathrm{s}} = 6\pi\eta r v,$$
  where r is the radius of the object, η is the fluid viscosity, and v is the object's velocity.

Conceptual Questions

Exercise:

Problem:

Athletes such as swimmers and bicyclists wear body suits in competition. Formulate a list of pros and cons of such suits.

Exercise:

Problem:

Two expressions were used for the drag force experienced by a moving object in a liquid. One depended upon the speed, while the other was proportional to the square of the speed. In which types of motion would each of these expressions be more applicable than the other one?

Exercise:

Problem:

As cars travel, oil and gasoline leaks onto the road surface. If a light rain falls, what does this do to the control of the car? Does a heavy rain make any difference?

Exercise:

Problem:

Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

Problems & Exercise

Exercise:

Problem:

The terminal velocity of a person falling in air depends upon the weight and the area of the person facing the fluid. Find the terminal velocity (in meters per second and kilometers per hour) of an 80.0-kg skydiver falling in a pike (headfirst) position with a surface area of $0.140\ \text{m}^2$.

Solution:

$115\ \text{m/s}$; $414\ \text{km/hr}$

Exercise:

Problem:

A 60-kg and a 90-kg skydiver jump from an airplane at an altitude of 6000 m, both falling in the pike position. Make some assumption on their frontal areas and calculate their terminal velocities. How long will it take for each skydiver to reach the ground (assuming the time to reach terminal velocity is small)? Assume all values are accurate to three significant digits.

Exercise:

Problem:

A 560-g squirrel with a surface area of $930\ \text{cm}^2$ falls from a 5.0-m tree to the ground. Estimate its terminal velocity. (Use a drag coefficient for a horizontal skydiver.) What will be the velocity of a 56-kg person hitting the ground, assuming no drag contribution in such a short distance?

Solution:

$25\ \text{m/s}$; $9.9\ \text{m/s}$

Exercise:

Problem:

To maintain a constant speed, the force provided by a car’s engine must equal the drag force plus the force of friction of the road (the rolling resistance). (a) What are the magnitudes of drag forces at 70 km/h and 100 km/h for a Toyota Camry? (Drag area is $0.70\ \text{m}^2$) (b) What is the magnitude of drag force at 70 km/h and 100 km/h for a Hummer H2? (Drag area is $2.44\ \text{m}^2$) Assume all values are accurate to three significant digits.

Exercise:

Problem:

By what factor does the drag force on a car increase as it goes from 65 to 110 km/h?

Solution:

2.9

Exercise:

Problem:

Calculate the speed a spherical rain drop would achieve falling from 5.00 km (a) in the absence of air drag (b) with air drag. Take the size across of the drop to be 4 mm, the density to be $1.00 \times 10^3\ \text{kg/m}^3$, and the surface area to be πr^2.

Exercise:

Problem:

Using Stokes’ law, verify that the units for viscosity are kilograms per meter per second.

Solution:
Equation:

$$[\eta] = \frac{[F_s]}{[r][v]} = \frac{\mathrm{kg} \cdot \mathrm{m/s}^2}{\mathrm{m} \cdot \mathrm{m/s}} = \frac{\mathrm{kg}}{\mathrm{m} \cdot \mathrm{s}}$$

Exercise:

Problem:

Find the terminal velocity of a spherical bacterium (diameter $2.00\ \mu\mathrm{m}$) falling in water. You will first need to note that the drag force is equal to the weight at terminal velocity. Take the density of the bacterium to be $1.10 \times 10^3\ \mathrm{kg/m}^3$.

Exercise:

Problem:

Stokes' law describes sedimentation of particles in liquids and can be used to measure viscosity. Particles in liquids achieve terminal velocity quickly. One can measure the time it takes for a particle to fall a certain distance and then use Stokes' law to calculate the viscosity of the liquid. Suppose a steel ball bearing (density $7.8 \times 10^3\ \mathrm{kg/m}^3$, diameter $3.0\ \mathrm{mm}$) is dropped in a container of motor oil. It takes 12 s to fall a distance of 0.60 m. Calculate the viscosity of the oil.

Solution:

$0.76\ \mathrm{kg/m} \cdot \mathrm{s}$

Glossary

drag force

F_D, found to be proportional to the square of the speed of the object; mathematically

Equation:

$$F_\mathrm{D} \propto v^2$$

Equation:

$$F_{\mathrm{D}} = \frac{1}{2} C\rho\, A v^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid

Stokes' law

$F_{\mathrm{s}} = 6\pi r\eta v$, where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity

Introduction to Uniform Circular Motion and Gravitation
class="introduction"
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This content is not covered in this course, and is here solely for your information.

This Australian Grand Prix Formula 1 race car moves in a circular path as it makes the turn. Its wheels also spin rapidly—the latter completing many revolutions, the former only part of one (a circular arc). The same physical principles

are involved in each. (credit: Richard Munckton)

Many motions, such as the arc of a bird’s flight or Earth’s path around the Sun, are curved. Recall that Newton’s first law tells us that motion is along a straight line at constant speed unless there is a net external force. We will therefore study not only motion along curves, but also the forces that cause it, including gravitational forces. In some ways, this chapter is a continuation of [Dynamics: Newton's Laws of Motion] as we study more applications of Newton’s laws of motion.

This chapter deals with the simplest form of curved motion, **uniform circular motion**, motion in a circular path at constant speed. Studying this topic illustrates most concepts associated with rotational motion and leads to the study of many new topics we group under the name *rotation*. Pure *rotational motion* occurs when points in an object move in circular paths centered on one point. Pure *translational motion* is motion with no rotation. Some motion combines both types, such as a rotating hockey puck moving along ice.

Glossary

uniform circular motion
: the motion of an object in a circular path at constant speed

Rotation Angle and Angular Velocity

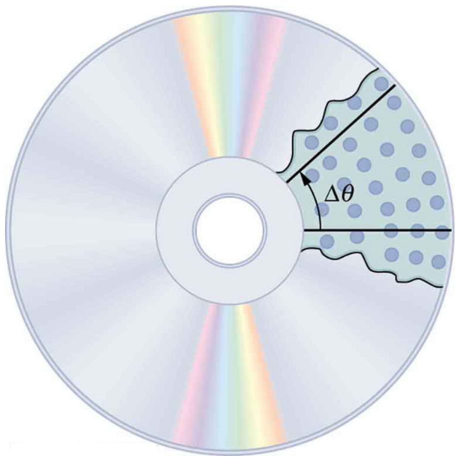
- Define arc length, rotation angle, radius of curvature and angular velocity.
- Calculate the angular velocity of a car wheel spin.

In Kinematics, we studied motion along a straight line and introduced such concepts as displacement, velocity, and acceleration. Two-Dimensional Kinematics dealt with motion in two dimensions. Projectile motion is a special case of two-dimensional kinematics in which the object is projected into the air, while being subject to the gravitational force, and lands a distance away. In this chapter, we consider situations where the object does not land but moves in a curve. We begin the study of uniform circular motion by defining two angular quantities needed to describe rotational motion.

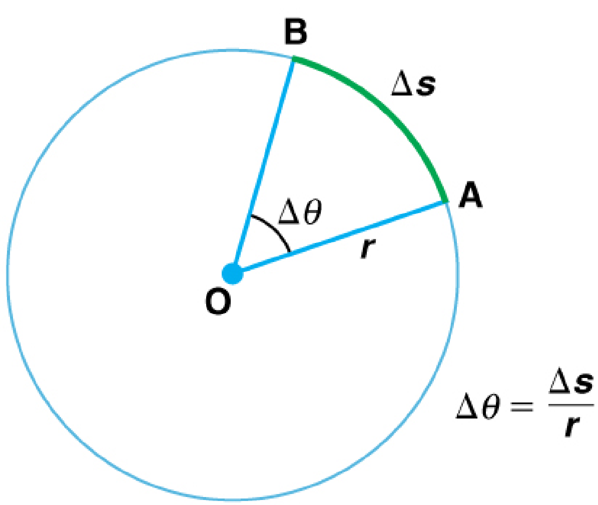
Rotation Angle

When objects rotate about some axis—for example, when the CD (compact disc) in [link] rotates about its center—each point in the object follows a circular arc. Consider a line from the center of the CD to its edge. Each **pit** used to record sound along this line moves through the same angle in the same amount of time. The rotation angle is the amount of rotation and is analogous to linear distance. We define the **rotation angle** $\Delta\theta$ to be the ratio of the arc length to the radius of curvature:
Equation:

$$\Delta\theta = \frac{\Delta s}{r}.$$



All points on a CD travel in circular arcs. The pits along a line from the center to the edge all move through the same angle $\Delta\theta$ in a time Δt.



The radius of a circle is rotated through an angle $\Delta\theta$. The arc

length Δs is described on the circumference.

The **arc length** Δs is the distance traveled along a circular path as shown in [link] Note that r is the **radius of curvature** of the circular path.

We know that for one complete revolution, the arc length is the circumference of a circle of radius r. The circumference of a circle is $2\pi r$. Thus for one complete revolution the rotation angle is

Equation:

$$\Delta\theta = \frac{2\pi r}{r} = 2\pi.$$

This result is the basis for defining the units used to measure rotation angles, $\Delta\theta$ to be **radians** (rad), defined so that

Equation:

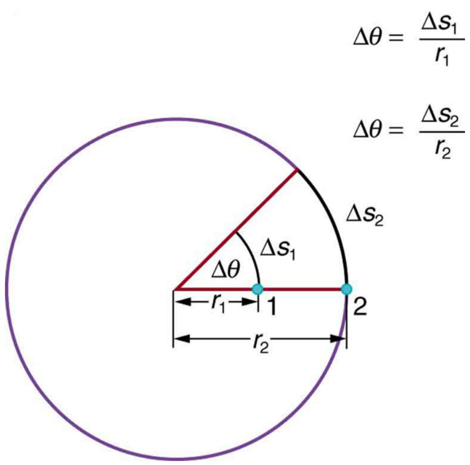
$$2\pi \text{ rad} = 1 \text{ revolution.}$$

A comparison of some useful angles expressed in both degrees and radians is shown in [link].

Degree Measures	Radian Measure
30º	$\frac{\pi}{6}$
60º	$\frac{\pi}{3}$

Degree Measures	Radian Measure
90°	$\frac{\pi}{2}$
120°	$\frac{2\pi}{3}$
135°	$\frac{3\pi}{4}$
180°	π

Comparison of Angular Units



Points 1 and 2 rotate through the same angle ($\Delta\theta$), but point 2 moves through a greater arc length (Δs) because it is at a greater distance from the center of rotation (r).

If $\Delta\theta = 2\pi$ rad, then the CD has made one complete revolution, and every point on the CD is back at its original position. Because there are 360^{o} in a circle or one revolution, the relationship between radians and degrees is thus

Equation:

$$2\pi \text{ rad} = 360^{\mathrm{o}}$$

so that

Equation:

$$1 \text{ rad} = \frac{360^{\mathrm{o}}}{2\pi} \approx 57.3^{\mathrm{o}}.$$

Angular Velocity

How fast is an object rotating? We define **angular velocity** ω as the rate of change of an angle. In symbols, this is

Equation:

$$\omega = \frac{\Delta\theta}{\Delta t},$$

where an angular rotation $\Delta\theta$ takes place in a time Δt. The greater the rotation angle in a given amount of time, the greater the angular velocity. The units for angular velocity are radians per second (rad/s).

Angular velocity ω is analogous to linear velocity v. To get the precise relationship between angular and linear velocity, we again consider a pit on the rotating CD. This pit moves an arc length Δs in a time Δt, and so it has a linear velocity

Equation:

$$v = \frac{\Delta s}{\Delta t}.$$

From $\Delta\theta = \frac{\Delta s}{r}$ we see that $\Delta s = r\Delta\theta$. Substituting this into the expression for v gives

Equation:

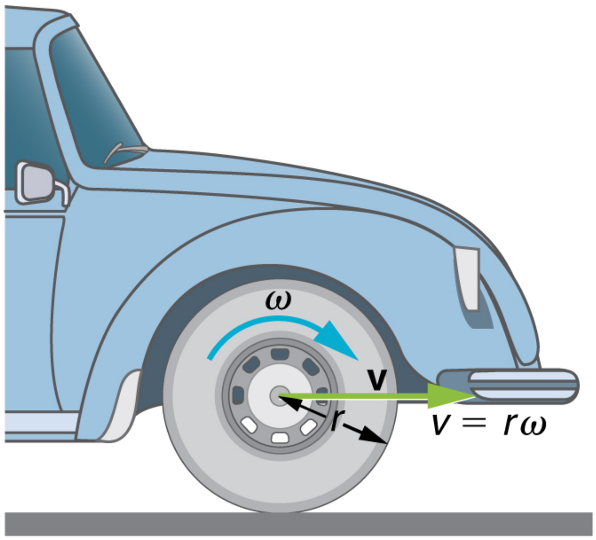
$$v = \frac{r\Delta\theta}{\Delta t} = r\omega.$$

We write this relationship in two different ways and gain two different insights:

Equation:

$$v = r\omega \text{ or } \omega = \frac{v}{r}.$$

The first relationship in $v = r\omega \text{ or } \omega = \frac{v}{r}$ states that the linear velocity v is proportional to the distance from the center of rotation, thus, it is largest for a point on the rim (largest r), as you might expect. We can also call this linear speed v of a point on the rim the *tangential speed*. The second relationship in $v = r\omega \text{ or } \omega = \frac{v}{r}$ can be illustrated by considering the tire of a moving car. Note that the speed of a point on the rim of the tire is the same as the speed v of the car. See [link]. So the faster the car moves, the faster the tire spins—large v means a large ω, because $v = r\omega$. Similarly, a larger-radius tire rotating at the same angular velocity (ω) will produce a greater linear speed (v) for the car.



A car moving at a velocity v to the right has a tire rotating with an angular velocity ω.The speed of the tread of the tire relative to the axle is v, the same as if the car were jacked up. Thus the car moves forward at linear velocity $v = r\omega$, where r is the tire radius. A larger angular velocity for the tire means a greater velocity for the car.

Example:
How Fast Does a Car Tire Spin?
Calculate the angular velocity of a $0.300\ \text{m}$ radius car tire when the car travels at $15.0\ \text{m/s}$ (about $54\ \text{km/h}$). See [link].
Strategy
Because the linear speed of the tire rim is the same as the speed of the car, we have $v = 15.0\ \text{m/s}$. The radius of the tire is given to be $r = 0.300\ \text{m}$.

Knowing v and r, we can use the second relationship in $v = r\omega$, $\omega = \frac{v}{r}$ to calculate the angular velocity.

Solution

To calculate the angular velocity, we will use the following relationship:

Equation:

$$\omega = \frac{v}{r}.$$

Substituting the knowns,

Equation:

$$\omega = \frac{15.0\ \text{m/s}}{0.300\ \text{m}} = 50.0\ \text{rad/s}.$$

Discussion

When we cancel units in the above calculation, we get 50.0/s. But the angular velocity must have units of rad/s. Because radians are actually unitless (radians are defined as a ratio of distance), we can simply insert them into the answer for the angular velocity. Also note that if an earth mover with much larger tires, say 1.20 m in radius, were moving at the same speed of 15.0 m/s, its tires would rotate more slowly. They would have an angular velocity

Equation:

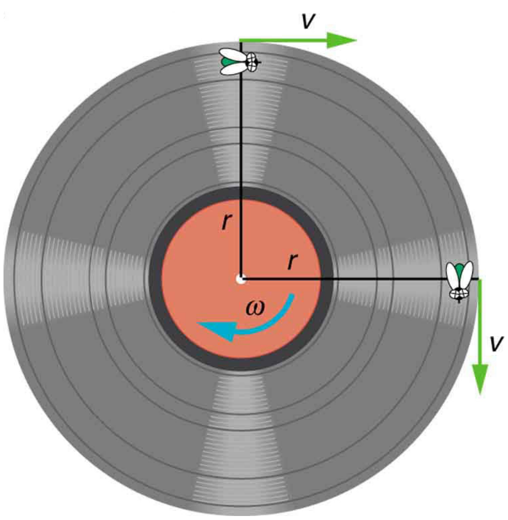
$$\omega = (15.0\ \text{m/s})/(1.20\ \text{m}) = 12.5\ \text{rad/s}.$$

Both ω and v have directions (hence they are angular and linear *velocities*, respectively). Angular velocity has only two directions with respect to the axis of rotation—it is either clockwise or counterclockwise. Linear velocity is tangent to the path, as illustrated in [link].

Note:

Take-Home Experiment

Tie an object to the end of a string and swing it around in a horizontal circle above your head (swing at your wrist). Maintain uniform speed as the object swings and measure the angular velocity of the motion. What is the approximate speed of the object? Identify a point close to your hand and take appropriate measurements to calculate the linear speed at this point. Identify other circular motions and measure their angular velocities.



As an object moves in a circle, here a fly on the edge of an old-fashioned vinyl record, its instantaneous velocity is always tangent to the circle. The direction of the angular velocity is clockwise in this case.

Note:
PhET Explorations: Ladybug Revolution

Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle, or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's x,y position, velocity, and acceleration using vectors or graphs. https://archive.cnx.org/specials/317a2b1e-2fbd-11e5-99b5-e38ffb545fe6/ladybug-motion/#sim-ladybug-motion

Section Summary

- Uniform circular motion is motion in a circle at constant speed. The rotation angle $\Delta\theta$ is defined as the ratio of the arc length to the radius of curvature:
  Equation:
  $$\Delta\theta = \frac{\Delta s}{r},$$
  where arc length Δs is distance traveled along a circular path and r is the radius of curvature of the circular path. The quantity $\Delta\theta$ is measured in units of radians (rad), for which
  Equation:
  $$2\pi \text{ rad} = 360^\circ = 1 \text{ revolution.}$$
- The conversion between radians and degrees is $1 \text{ rad} = 57.3^\circ$.
- Angular velocity ω is the rate of change of an angle,
  Equation:
  $$\omega = \frac{\Delta\theta}{\Delta t},$$
  where a rotation $\Delta\theta$ takes place in a time Δt. The units of angular velocity are radians per second (rad/s). Linear velocity v and angular velocity ω are related by

Equation:

$$v = r\omega \text{ or } \omega = \frac{v}{r}.$$

Conceptual Questions

Exercise:

Problem:

There is an analogy between rotational and linear physical quantities. What rotational quantities are analogous to distance and velocity?

Problem Exercises

Exercise:

Problem:

Semi-trailer trucks have an odometer on one hub of a trailer wheel. The hub is weighted so that it does not rotate, but it contains gears to count the number of wheel revolutions—it then calculates the distance traveled. If the wheel has a 1.15 m diameter and goes through 200,000 rotations, how many kilometers should the odometer read?

Solution:

723 km

Exercise:

Problem:

Microwave ovens rotate at a rate of about 6 rev/min. What is this in revolutions per second? What is the angular velocity in radians per second?

Exercise:

Problem:

An automobile with 0.260 m radius tires travels 80,000 km before wearing them out. How many revolutions do the tires make, neglecting any backing up and any change in radius due to wear?

Solution:

$5 \times 10^7 \text{ rotations}$

Exercise:

Problem:

(a) What is the period of rotation of Earth in seconds? (b) What is the angular velocity of Earth? (c) Given that Earth has a radius of $6.4 \times 10^6 \text{ m}$ at its equator, what is the linear velocity at Earth's surface?

Exercise:

Problem:

A baseball pitcher brings his arm forward during a pitch, rotating the forearm about the elbow. If the velocity of the ball in the pitcher's hand is 35.0 m/s and the ball is 0.300 m from the elbow joint, what is the angular velocity of the forearm?

Solution:

117 rad/s

Exercise:

Problem:

In lacrosse, a ball is thrown from a net on the end of a stick by rotating the stick and forearm about the elbow. If the angular velocity of the ball about the elbow joint is 30.0 rad/s and the ball is 1.30 m from the elbow joint, what is the velocity of the ball?

Exercise:

Problem:

A truck with 0.420-m-radius tires travels at 32.0 m/s. What is the angular velocity of the rotating tires in radians per second? What is this in rev/min?

Solution:

76.2 rad/s

728 rpm

Exercise:

Problem:

Integrated Concepts When kicking a football, the kicker rotates his leg about the hip joint.

(a) If the velocity of the tip of the kicker's shoe is 35.0 m/s and the hip joint is 1.05 m from the tip of the shoe, what is the shoe tip's angular velocity?

(b) The shoe is in contact with the initially stationary 0.500 kg football for 20.0 ms. What average force is exerted on the football to give it a velocity of 20.0 m/s?

(c) Find the maximum range of the football, neglecting air resistance.

Solution:

(a) 33.3 rad/s

(b) 500 N

(c) 40.8 m

Exercise:

Problem: **Construct Your Own Problem**

Consider an amusement park ride in which participants are rotated about a vertical axis in a cylinder with vertical walls. Once the angular velocity reaches its full value, the floor drops away and friction between the walls and the riders prevents them from sliding down. Construct a problem in which you calculate the necessary angular velocity that assures the riders will not slide down the wall. Include a free body diagram of a single rider. Among the variables to consider are the radius of the cylinder and the coefficients of friction between the riders' clothing and the wall.

Glossary

arc length
: Δs, the distance traveled by an object along a circular path

pit
: a tiny indentation on the spiral track moulded into the top of the polycarbonate layer of CD

rotation angle
: the ratio of the arc length to the radius of curvature on a circular path:

$$\Delta\theta = \frac{\Delta s}{r}$$

radius of curvature
: radius of a circular path

radians
: a unit of angle measurement

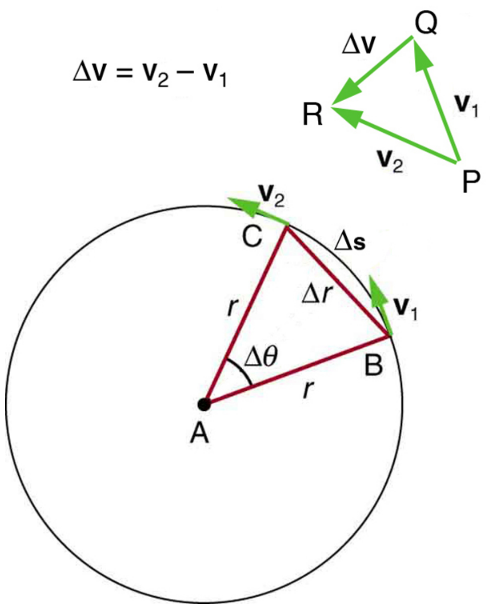
angular velocity
: ω, the rate of change of the angle with which an object moves on a circular path

Centripetal Acceleration

- Establish the expression for centripetal acceleration.
- Explain the centrifuge.

We know from kinematics that acceleration is a change in velocity, either in its magnitude or in its direction, or both. In uniform circular motion, the direction of the velocity changes constantly, so there is always an associated acceleration, even though the magnitude of the velocity might be constant. You experience this acceleration yourself when you turn a corner in your car. (If you hold the wheel steady during a turn and move at constant speed, you are in uniform circular motion.) What you notice is a sideways acceleration because you and the car are changing direction. The sharper the curve and the greater your speed, the more noticeable this acceleration will become. In this section we examine the direction and magnitude of that acceleration.

[link] shows an object moving in a circular path at constant speed. The direction of the instantaneous velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity, which points directly toward the center of rotation (the center of the circular path). This pointing is shown with the vector diagram in the figure. We call the acceleration of an object moving in uniform circular motion (resulting from a net external force) the **centripetal acceleration**(a_c); centripetal means “toward the center” or “center seeking.”



The directions of the velocity of an object at two different points are shown, and the change in velocity $\mathbf{\Delta v}$ is seen to point directly toward the center of curvature. (See small inset.) Because $\mathbf{a}_{\text{c}} = \mathbf{\Delta v}/\Delta t$, the acceleration is also toward the center; $\mathbf{a}_c$ is called centripetal acceleration. (Because $\Delta\theta$ is very small, the arc length Δs is equal to the chord length Δr for small time differences.)

The direction of centripetal acceleration is toward the center of curvature, but what is its magnitude? Note that the triangle formed by the velocity vectors and the one formed by the radii r and Δs are similar. Both the

triangles ABC and PQR are isosceles triangles (two equal sides). The two equal sides of the velocity vector triangle are the speeds $v_1 = v_2 = v$. Using the properties of two similar triangles, we obtain

Equation:

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}.$$

Acceleration is $\Delta v / \Delta t$, and so we first solve this expression for Δv:

Equation:

$$\Delta v = \frac{v}{r} \Delta s.$$

Then we divide this by Δt, yielding

Equation:

$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \times \frac{\Delta s}{\Delta t}.$$

Finally, noting that $\Delta v / \Delta t = a_{\mathrm{c}}$ and that $\Delta s / \Delta t = v$, the linear or tangential speed, we see that the magnitude of the centripetal acceleration is

Equation:

$$a_{\mathrm{c}} = \frac{v^2}{r},$$

which is the acceleration of an object in a circle of radius r at a speed v. So, centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you have noticed when driving a car. But it is a bit surprising that a_{c} is proportional to speed squared, implying, for example, that it is four times as hard to take a curve at 100 km/h than at 50 km/h. A sharp corner has a small radius, so that a_{c} is greater for tighter turns, as you have probably noticed.

It is also useful to express a_c in terms of angular velocity. Substituting $v = r\omega$ into the above expression, we find $a_c = (r\omega)^2/r = r\omega^2$. We can express the magnitude of centripetal acceleration using either of two equations:

Equation:

$$a_c = \frac{v^2}{r};\ a_c = r\omega^2.$$

Recall that the direction of a_c is toward the center. You may use whichever expression is more convenient, as illustrated in examples below.

A **centrifuge** (see [link]b) is a rotating device used to separate specimens of different densities. High centripetal acceleration significantly decreases the time it takes for separation to occur, and makes separation possible with small samples. Centrifuges are used in a variety of applications in science and medicine, including the separation of single cell suspensions such as bacteria, viruses, and blood cells from a liquid medium and the separation of macromolecules, such as DNA and protein, from a solution. Centrifuges are often rated in terms of their centripetal acceleration relative to acceleration due to gravity (g); maximum centripetal acceleration of several hundred thousand g is possible in a vacuum. Human centrifuges, extremely large centrifuges, have been used to test the tolerance of astronauts to the effects of accelerations larger than that of Earth's gravity.

Example:
How Does the Centripetal Acceleration of a Car Around a Curve Compare with That Due to Gravity?

What is the magnitude of the centripetal acceleration of a car following a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h)? Compare the acceleration with that due to gravity for this fairly gentle curve taken at highway speed. See [link](a).

Strategy

Because v and r are given, the first expression in $a_c = \frac{v^2}{r}; a_c = r\omega^2$ is the most convenient to use.

Solution

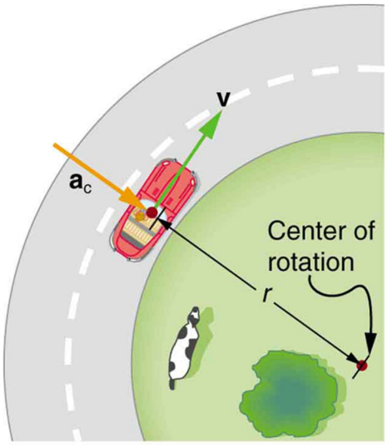
Entering the given values of $v = 25.0 \text{ m/s}$ and $r = 500 \text{ m}$ into the first expression for a_c gives

Equation:

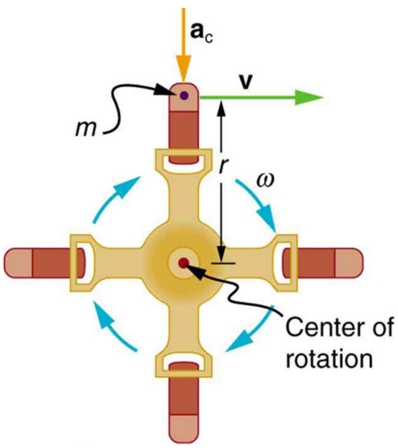
$$a_c = \frac{v^2}{r} = \frac{(25.0 \text{ m/s})^2}{500 \text{ m}} = 1.25 \text{ m/s}^2.$$

Discussion

To compare this with the acceleration due to gravity $(g = 9.80 \text{ m/s}^2)$, we take the ratio of $a_c/g = \left(1.25 \text{ m/s}^2\right)/\left(9.80 \text{ m/s}^2\right) = 0.128$. Thus, $a_c = 0.128$ g and is noticeable especially if you were not wearing a seat belt.



(a) Car around corner



(b) Centrifuge

(a) The car following a circular path at constant speed is accelerated perpendicular to its velocity, as shown. The magnitude of this centripetal acceleration is found in [link]. (b) A particle of mass in a centrifuge is rotating at constant angular velocity . It

must be accelerated perpendicular to its velocity or it would continue in a straight line. The magnitude of the necessary acceleration is found in [link].

Example:
How Big Is the Centripetal Acceleration in an Ultracentrifuge?
Calculate the centripetal acceleration of a point 7.50 cm from the axis of an **ultracentrifuge** spinning at $7.5 \times 10^4\ \text{rev/min}$. Determine the ratio of this acceleration to that due to gravity. See [link](b).
Strategy
The term rev/min stands for revolutions per minute. By converting this to radians per second, we obtain the angular velocity ω. Because r is given, we can use the second expression in the equation $a_c = \frac{v^2}{r};\ a_c = r\omega^2$ to calculate the centripetal acceleration.
Solution
To convert $7.50 \times 10^4\ \text{rev/min}$ to radians per second, we use the facts that one revolution is $2\pi\text{rad}$ and one minute is 60.0 s. Thus,
Equation:

$$\omega = 7.50 \times 10^4 \frac{\text{rev}}{\text{min}} \times \frac{2\pi\ \text{rad}}{1\ \text{rev}} \times \frac{1\ \text{min}}{60.0\ \text{s}} = 7854\ \text{rad/s}.$$

Now the centripetal acceleration is given by the second expression in $a_c = \frac{v^2}{r};\ a_c = r\omega^2$ as
Equation:

$$a_c = r\omega^2.$$

Converting 7.50 cm to meters and substituting known values gives
Equation:

$$a_c = (0.0750\ \text{m})(7854\ \text{rad/s})^2 = 4.63 \times 10^6\ \text{m/s}^2.$$

Note that the unitless radians are discarded in order to get the correct units for centripetal acceleration. Taking the ratio of a_c to g yields
Equation:

$$\frac{a_c}{g} = \frac{4.63 \times 10^6}{9.80} = 4.72 \times 10^5.$$

Discussion
This last result means that the centripetal acceleration is 472,000 times as strong as g. It is no wonder that such high ω centrifuges are called ultracentrifuges. The extremely large accelerations involved greatly decrease the time needed to cause the sedimentation of blood cells or other materials.

Of course, a net external force is needed to cause any acceleration, just as Newton proposed in his second law of motion. So a net external force is needed to cause a centripetal acceleration. In Centripetal Force, we will consider the forces involved in circular motion.

Note:
PhET Explorations: Ladybug Motion 2D
Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.
https://archive.cnx.org/specials/317a2b1e-2fbd-11e5-99b5-e38ffb545fe6/ladybug-motion/#sim-ladybug-motion

Section Summary

- Centripetal acceleration a_{c} is the acceleration experienced while in uniform circular motion. It always points toward the center of rotation. It is perpendicular to the linear velocity v and has the magnitude **Equation:**

$$a_{\mathrm{c}} = \frac{v^2}{r}; a_{\mathrm{c}} = r\omega^2.$$

- The unit of centripetal acceleration is $\mathrm{m/s^2}$.

Conceptual Questions

Exercise:

Problem:

Can centripetal acceleration change the speed of circular motion? Explain.

Problem Exercises

Exercise:

Problem:

A fairground ride spins its occupants inside a flying saucer-shaped container. If the horizontal circular path the riders follow has an 8.00 m radius, at how many revolutions per minute will the riders be subjected to a centripetal acceleration whose magnitude is 1.50 times that due to gravity?

Solution:

12.9 rev/min

Exercise:

Problem:

A runner taking part in the 200 m dash must run around the end of a track that has a circular arc with a radius of curvature of 30 m. If he completes the 200 m dash in 23.2 s and runs at constant speed throughout the race, what is the magnitude of his centripetal acceleration as he runs the curved portion of the track?

Exercise:

Problem:

Taking the age of Earth to be about 4×10^9 years and assuming its orbital radius of 1.5×10^{11} m has not changed and is circular, calculate the approximate total distance Earth has traveled since its birth (in a frame of reference stationary with respect to the Sun).

Solution:

4×10^{21} m

Exercise:

Problem:

The propeller of a World War II fighter plane is 2.30 m in diameter.

(a) What is its angular velocity in radians per second if it spins at 1200 rev/min?

(b) What is the linear speed of its tip at this angular velocity if the plane is stationary on the tarmac?

(c) What is the centripetal acceleration of the propeller tip under these conditions? Calculate it in meters per second squared and convert to multiples of *g*.

Exercise:

Problem:

An ordinary workshop grindstone has a radius of 7.50 cm and rotates at 6500 rev/min.

(a) Calculate the magnitude of the centripetal acceleration at its edge in meters per second squared and convert it to multiples of g.

(b) What is the linear speed of a point on its edge?

Solution:

a) $3.47 \times 10^{4}\ \mathrm{m/s^{2}}$, $3.55 \times 10^{3}\ g$

b) $51.1\ \mathrm{m/s}$

Exercise:

Problem:

Helicopter blades withstand tremendous stresses. In addition to supporting the weight of a helicopter, they are spun at rapid rates and experience large centripetal accelerations, especially at the tip.

(a) Calculate the magnitude of the centripetal acceleration at the tip of a 4.00 m long helicopter blade that rotates at 300 rev/min.

(b) Compare the linear speed of the tip with the speed of sound (taken to be 340 m/s).

Exercise:

Problem: Olympic ice skaters are able to spin at about 5 rev/s.

(a) What is their angular velocity in radians per second?

(b) What is the centripetal acceleration of the skater’s nose if it is 0.120 m from the axis of rotation?

(c) An exceptional skater named Dick Button was able to spin much faster in the 1950s than anyone since—at about 9 rev/s. What was the centripetal acceleration of the tip of his nose, assuming it is at 0.120 m radius?

(d) Comment on the magnitudes of the accelerations found. It is reputed that Button ruptured small blood vessels during his spins.

Solution:

a) 31.4 rad/s

b) 118 m/s

c) 384 m/s

d)The centripetal acceleration felt by Olympic skaters is 12 times larger than the acceleration due to gravity. That's quite a lot of acceleration in itself. The centripetal acceleration felt by Button's nose was 39.2 times larger than the acceleration due to gravity. It is no wonder that he ruptured small blood vessels in his spins.

Exercise:

Problem:

What percentage of the acceleration at Earth's surface is the acceleration due to gravity at the position of a satellite located 300 km above Earth?

Exercise:

Problem:

Verify that the linear speed of an ultracentrifuge is about 0.50 km/s, and Earth in its orbit is about 30 km/s by calculating:

(a) The linear speed of a point on an ultracentrifuge 0.100 m from its center, rotating at 50,000 rev/min.

(b) The linear speed of Earth in its orbit about the Sun (use data from the text on the radius of Earth’s orbit and approximate it as being circular).

Solution:

a) 0.524 km/s

b) 29.7 km/s

Exercise:

Problem:

A rotating space station is said to create “artificial gravity”—a loosely-defined term used for an acceleration that would be crudely similar to gravity. The outer wall of the rotating space station would become a floor for the astronauts, and centripetal acceleration supplied by the floor would allow astronauts to exercise and maintain muscle and bone strength more naturally than in non-rotating space environments. If the space station is 200 m in diameter, what angular velocity would produce an “artificial gravity” of $9.80\ \text{m/s}^2$ at the rim?

Exercise:

Problem:

At takeoff, a commercial jet has a 60.0 m/s speed. Its tires have a diameter of 0.850 m.

(a) At how many rev/min are the tires rotating?

(b) What is the centripetal acceleration at the edge of the tire?

(c) With what force must a determined 1.00×10^{-15} kg bacterium cling to the rim?

(d) Take the ratio of this force to the bacterium’s weight.

Solution:

(a) $1.35 \times 10^3\ \text{rpm}$

(b) $8.47 \times 10^3\ \text{m/s}^2$

(c) $8.47 \times 10^{-12}\ \text{N}$

(d) 865

Exercise:

Problem: **Integrated Concepts**

Riders in an amusement park ride shaped like a Viking ship hung from a large pivot are rotated back and forth like a rigid pendulum. Sometime near the middle of the ride, the ship is momentarily motionless at the top of its circular arc. The ship then swings down under the influence of gravity.

(a) Assuming negligible friction, find the speed of the riders at the bottom of its arc, given the system's center of mass travels in an arc having a radius of 14.0 m and the riders are near the center of mass.

(b) What is the centripetal acceleration at the bottom of the arc?

(c) Draw a free body diagram of the forces acting on a rider at the bottom of the arc.

(d) Find the force exerted by the ride on a 60.0 kg rider and compare it to her weight.

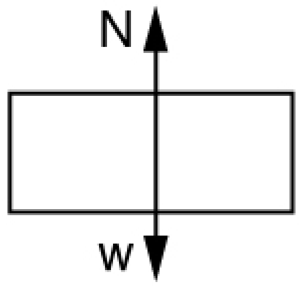
(e) Discuss whether the answer seems reasonable.

Solution:

(a) $16.6\ \text{m/s}$

(b) $19.6\ \text{m/s}^2$

(c)



(d) 1.76×10^3 N or $3.00\ w$, that is, the normal force (upward) is three times her weight.

(e) This answer seems reasonable, since she feels like she’s being forced into the chair MUCH stronger than just by gravity.

Exercise:

Problem: Unreasonable Results

A mother pushes her child on a swing so that his speed is 9.00 m/s at the lowest point of his path. The swing is suspended 2.00 m above the child’s center of mass.

(a) What is the magnitude of the centripetal acceleration of the child at the low point?

(b) What is the magnitude of the force the child exerts on the seat if his mass is 18.0 kg?

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent?

Solution:

a) $40.5\ \mathrm{m/s^2}$

b) 905 N

c) The force in part (b) is very large. The acceleration in part (a) is too much, about 4 g.

d) The speed of the swing is too large. At the given velocity at the bottom of the swing, there is enough kinetic energy to send the child all the way over the top, ignoring friction.

Glossary

centripetal acceleration
: the acceleration of an object moving in a circle, directed toward the center

ultracentrifuge
: a centrifuge optimized for spinning a rotor at very high speeds

Centripetal Force

By the end of the section, you will be able to:

- Explain the equation for centripetal acceleration
- Apply Newton's second law to develop the equation for centripetal force
- Use circular motion concepts in solving problems involving Newton's laws of motion

In Motion in Two and Three Dimensions, we examined the basic concepts of circular motion. An object undergoing circular motion, like one of the race cars shown at the beginning of this chapter, must be accelerating because it is changing the direction of its velocity. We proved that this centrally directed acceleration, called centripetal acceleration, is given by the formula

Equation:

$$a_{\mathrm{c}}=\frac{v^2}{r}$$

where *v* is the velocity of the object, directed along a tangent line to the curve at any instant. If we know the angular velocity ω, then we can use

Equation:

$$a_{\mathrm{c}}=r\omega^2.$$

Angular velocity gives the rate at which the object is turning through the curve, in units of rad/s. This acceleration acts along the radius of the curved path and is thus also referred to as a radial acceleration.

An acceleration must be produced by a force. Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge. Any net force causing uniform circular motion is called a **centripetal force**. The direction

of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law of motion, net force is mass times acceleration: $F_{\text{net}} = ma$. For uniform circular motion, the acceleration is the centripetal acceleration: $a = a_{\text{c}}$. Thus, the magnitude of centripetal force F_{c} is

Equation:

$$F_{\text{c}} = ma_{\text{c}}.$$

By substituting the expressions for centripetal acceleration a_{c} $\left(a_{\text{c}} = \frac{v^2}{r}; a_{\text{c}} = r\omega^2\right)$, we get two expressions for the centripetal force F_{c} in terms of mass, velocity, angular velocity, and radius of curvature:

Note:

Equation:

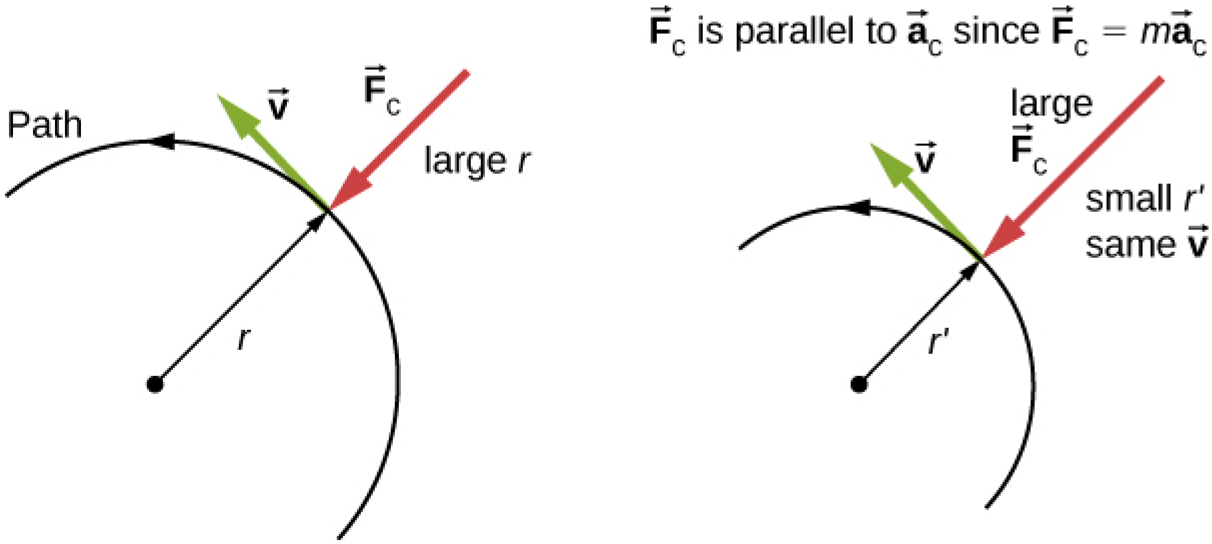
$$F_{\text{c}} = m\frac{v^2}{r}; \quad F_{\text{c}} = mr\omega^2.$$

You may use whichever expression for centripetal force is more convenient. Centripetal force $\vec{\mathbf{F}}_{\text{c}}$ is always perpendicular to the path and points to the center of curvature, because $\vec{\mathbf{a}}_{\text{c}}$ is perpendicular to the velocity and points to the center of curvature. Note that if you solve the first expression for *r*, you get

Equation:

$$r = \frac{mv^2}{F_{\text{c}}}.$$

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve, as in [link].

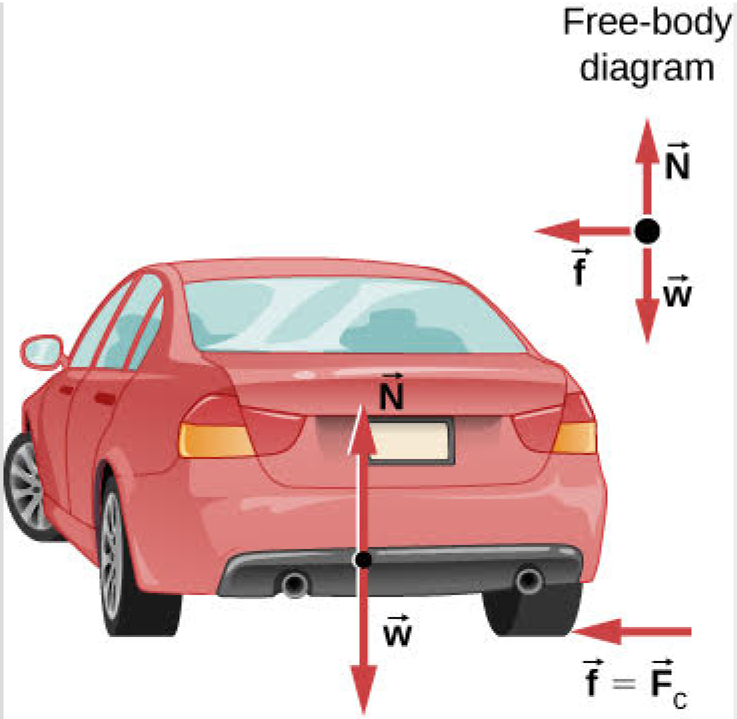


The frictional force supplies the centripetal force and is numerically equal to it. Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the F_c, the smaller the radius of curvature r and the sharper the curve. The second curve has the same v, but a larger F_c produces a smaller r'.

Example:

What Coefficient of Friction Do Cars Need on a Flat Curve?

(a) Calculate the centripetal force exerted on a 900.0-kg car that negotiates a 500.0-m radius curve at 25.00 m/s. (b) Assuming an unbanked curve, find the minimum static coefficient of friction between the tires and the road, static friction being the reason that keeps the car from slipping ([link]).



This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.

Strategy

a. We know that $F_c = \frac{mv^2}{r}$. Thus,
   Equation:

$$F_c = \frac{mv^2}{r} = \frac{(900.0\,\text{kg})(25.00\,\text{m/s})^2}{(500.0\,\text{m})} = 1125\,\text{N}.$$

b. [link] shows the forces acting on the car on an unbanked (level ground) curve. Friction is to the left, keeping the car from slipping,

and because it is the only horizontal force acting on the car, the friction is the centripetal force in this case. We know that the maximum static friction (at which the tires roll but do not slip) is $\mu_s N$, where μ_s is the static coefficient of friction and N is the normal force. The normal force equals the car's weight on level ground, so $N = mg$. Thus the centripetal force in this situation is

Equation:

$$F_c \equiv f = \mu_s N = \mu_s mg.$$

Now we have a relationship between centripetal force and the coefficient of friction. Using the equation

Equation:

$$F_c = m\frac{v^2}{r},$$

we obtain

Equation:

$$m\frac{v^2}{r} = \mu_s mg.$$

We solve this for μ_s, noting that mass cancels, and obtain

Equation:

$$\mu_s = \frac{v^2}{rg}.$$

Substituting the knowns,

Equation:

$$\mu_s = \frac{(25.00\,\text{m/s})^2}{(500.0\,\text{m})(9.80\,\text{m/s}^2)} = 0.13.$$

(Because coefficients of friction are approximate, the answer is given to only two digits.)

Significance

The coefficient of friction found in [link](b) is much smaller than is typically found between tires and roads. The car still negotiates the curve if the coefficient is greater than 0.13, because static friction is a responsive force, able to assume a value less than but no more than $\mu_s N$. A higher coefficient would also allow the car to negotiate the curve at a higher speed, but if the coefficient of friction is less, the safe speed would be less than 25 m/s. Note that mass cancels, implying that, in this example, it does not matter how heavily loaded the car is to negotiate the turn. Mass cancels because friction is assumed proportional to the normal force, which in turn is proportional to mass. If the surface of the road were banked, the normal force would be greater, as discussed next.

Note:
Exercise:

Problem:

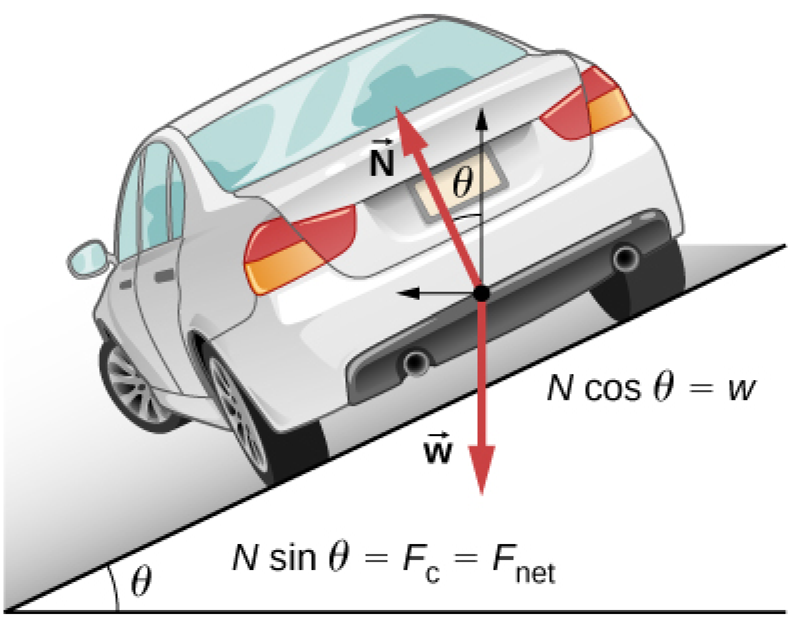
Check Your Understanding A car moving at 96.8 km/h travels around a circular curve of radius 182.9 m on a flat country road. What must be the minimum coefficient of static friction to keep the car from slipping?

Solution:

0.40

Banked Curves

Let us now consider **banked curves**, where the slope of the road helps you negotiate the curve ([link]). The greater the angle θ, the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an "ideally banked curve," the angle θ is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. We will derive an expression for θ for an ideally banked curve and consider an example related to it.



The car on this banked curve is moving away and turning to the left.

For **ideal banking**, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force N in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In cases in which forces are not parallel, it is most convenient to consider components along perpendicular axes—in this case, the vertical and horizontal directions.

[link] shows a free-body diagram for a car on a frictionless banked curve. If the angle θ is ideal for the speed and radius, then the net external force equals the necessary centripetal force. The only two external forces acting on the car are its weight $\vec{\mathbf{w}}$ and the normal force of the road $\vec{\mathbf{N}}$. (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude mv^2/r. Because this is the crucial force and it is horizontal, we use a coordinate system with vertical and horizontal axes. Only the normal force has a horizontal component, so this must equal the centripetal force, that is,

Equation:

$$N\sin\theta = \frac{mv^2}{r}.$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From [link], we see that the vertical component of the normal force is $N\cos\theta$, and the only other vertical force is the car's weight. These must be equal in magnitude; thus,

Equation:

$$N\cos\theta = mg.$$

Now we can combine these two equations to eliminate N and get an expression for θ, as desired. Solving the second equation for $N = mg/\left(cos\theta\right)$ and substituting this into the first yields

Equation:

$$\begin{array}{rcl} mg\frac{\sin\theta}{\cos\theta} & = & \frac{mv^2}{r} \\ mg\tan\theta & = & \frac{mv^2}{r} \\ \tan\theta & = & \frac{v^2}{rg}. \end{array}$$

Taking the inverse tangent gives

Note:
Equation:

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right).$$

This expression can be understood by considering how θ depends on *v* and *r*. A large θ is obtained for a large *v* and a small *r*. That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve were frictionless. Note that θ does not depend on the mass of the vehicle.

Example:
What Is the Ideal Speed to Take a Steeply Banked Tight Curve?
Curves on some test tracks and race courses, such as Daytona International Speedway in Florida, are very steeply banked. This banking, with the aid of tire friction and very stable car configurations, allows the curves to be taken at very high speed. To illustrate, calculate the speed at which a 100.0-m radius curve banked at 31.0° should be driven if the road were frictionless.
Strategy
We first note that all terms in the expression for the ideal angle of a banked curve except for speed are known; thus, we need only rearrange it so that speed appears on the left-hand side and then substitute known quantities.
Solution
Starting with
Equation:

$$\tan\theta = \frac{v^2}{rg},$$

we get

Equation:

$$v = \sqrt{rg\tan\theta}.$$

Noting that $\tan 31.0^\circ = 0.609$, we obtain

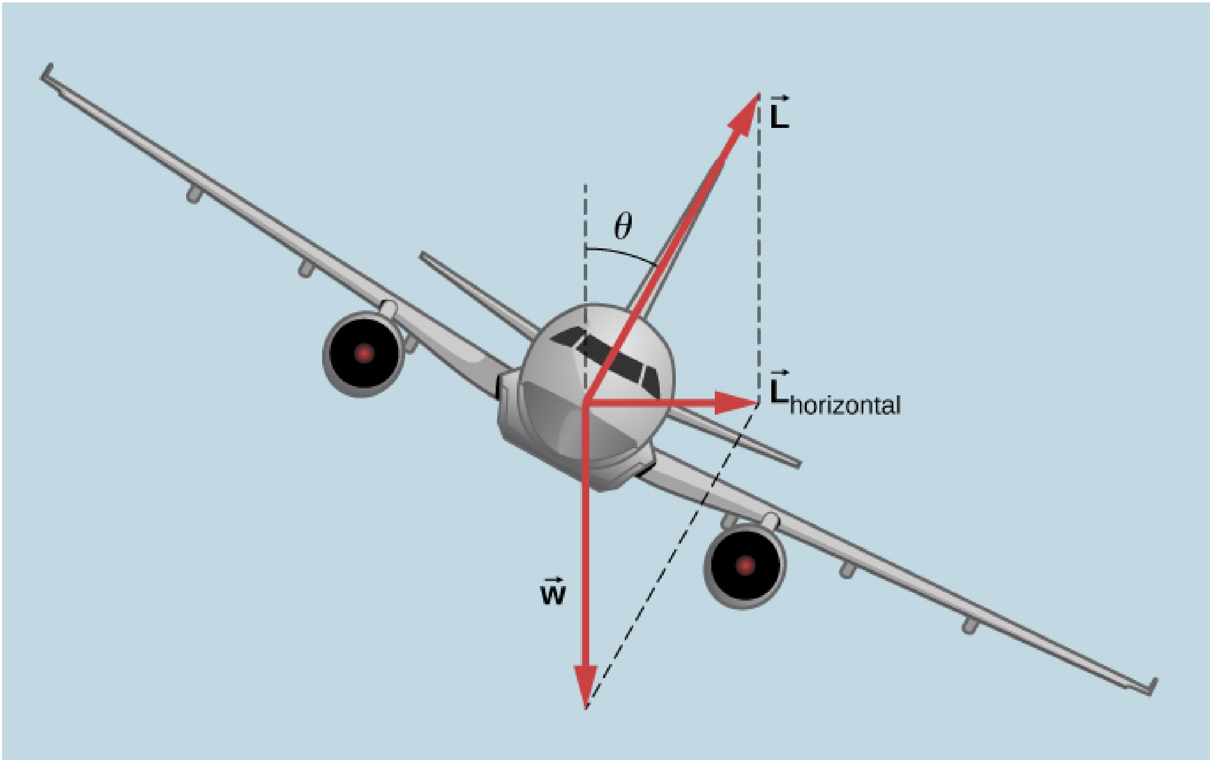
Equation:

$$v = \sqrt{(100.0\,\text{m})(9.80\,\text{m/s}^2)(0.609)} = 24.4\,\text{m/s}.$$

Significance

This is just about 165 km/h, consistent with a very steeply banked and rather sharp curve. Tire friction enables a vehicle to take the curve at significantly higher speeds.

Airplanes also make turns by banking. The lift force, due to the force of the air on the wing, acts at right angles to the wing. When the airplane banks, the pilot is obtaining greater lift than necessary for level flight. The vertical component of lift balances the airplane’s weight, and the horizontal component accelerates the plane. The banking angle shown in [link] is given by θ. We analyze the forces in the same way we treat the case of the car rounding a banked curve.



In a banked turn, the horizontal component of lift is unbalanced and accelerates the plane. The normal component of lift balances the plane's weight. The banking angle is given by θ. Compare the vector diagram with that shown in [link].

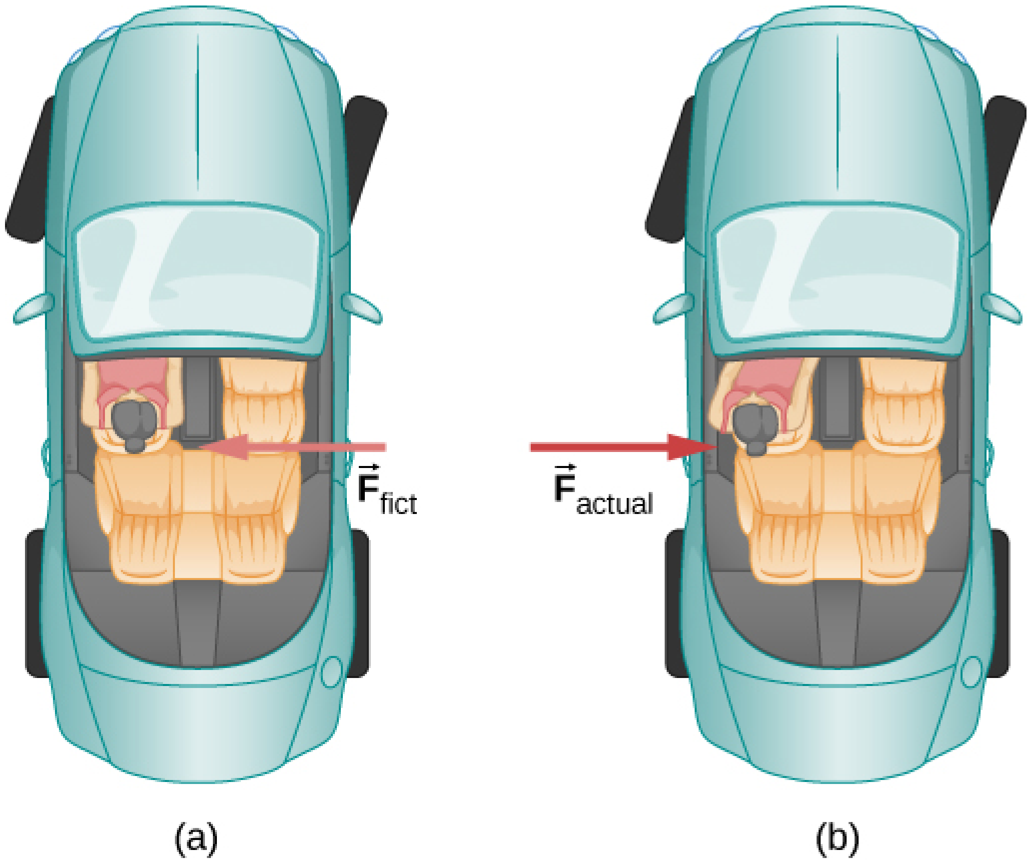
Note:
Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's *xy*-position, velocity, and acceleration using vectors or graphs.

Note:
A circular motion requires a force, the so-called centripetal force, which is directed to the axis of rotation. This simplified model of a carousel

demonstrates this force.

Inertial Forces and Noninertial (Accelerated) Frames: The Coriolis Force

What do taking off in a jet airplane, turning a corner in a car, riding a merry-go-round, and the circular motion of a tropical cyclone have in common? Each exhibits inertial forces—forces that merely seem to arise from motion, because the observer's frame of reference is accelerating or rotating. When taking off in a jet, most people would agree it feels as if you are being pushed back into the seat as the airplane accelerates down the runway. Yet a physicist would say that *you* tend to remain stationary while the *seat* pushes forward on you. An even more common experience occurs when you make a tight curve in your car—say, to the right ([link]). You feel as if you are thrown (that is, *forced*) toward the left relative to the car. Again, a physicist would say that *you* are going in a straight line (recall Newton's first law) but the *car* moves to the right, not that you are experiencing a force from the left.



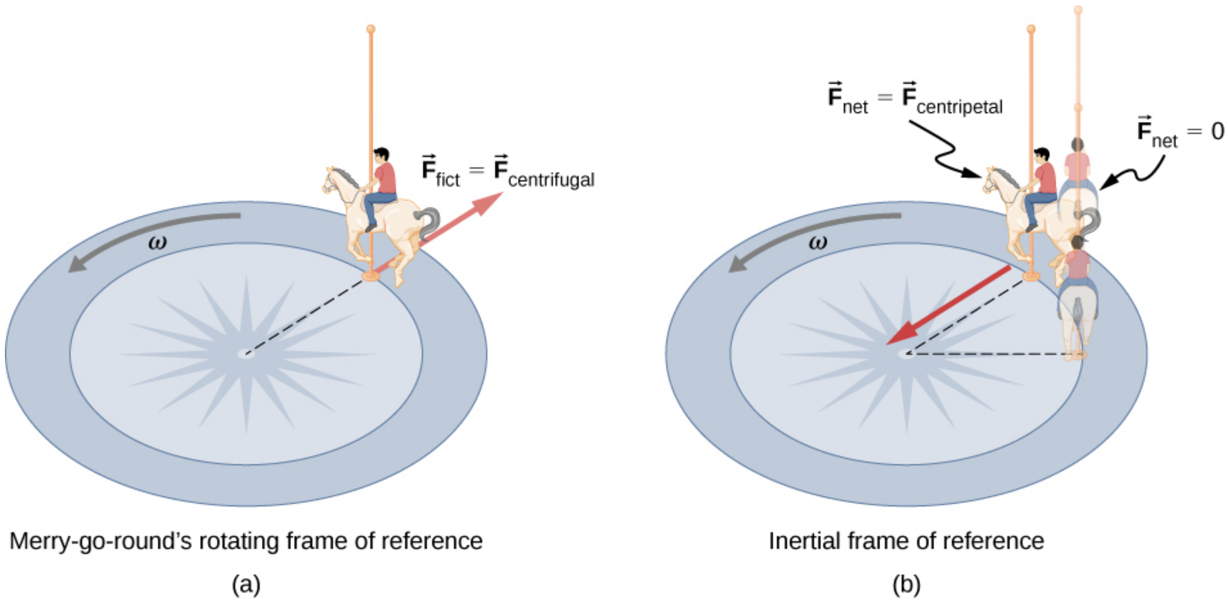
(a) The car driver feels herself forced to the left relative to the car when she makes a right turn. This is an inertial force arising from the use of the car as a frame of reference. (b) In Earth's frame of reference, the driver moves in a straight line, obeying Newton's first law, and the car moves to the right. There is no force to the left on the driver relative to Earth. Instead, there is a force to the right on the car to make it turn.

We can reconcile these points of view by examining the frames of reference used. Let us concentrate on people in a car. Passengers instinctively use the car as a frame of reference, whereas a physicist might use Earth. The physicist might make this choice because Earth is nearly an inertial frame of reference, in which all forces have an identifiable physical origin. In such a frame of reference, Newton's laws of motion take the form given in

Newton’s Laws of Motion. The car is a **noninertial frame of reference** because it is accelerated to the side. The force to the left sensed by car passengers is an **inertial force** having no physical origin (it is due purely to the inertia of the passenger, not to some physical cause such as tension, friction, or gravitation). The car, as well as the driver, is actually accelerating to the right. This inertial force is said to be an inertial force because it does not have a physical origin, such as gravity.

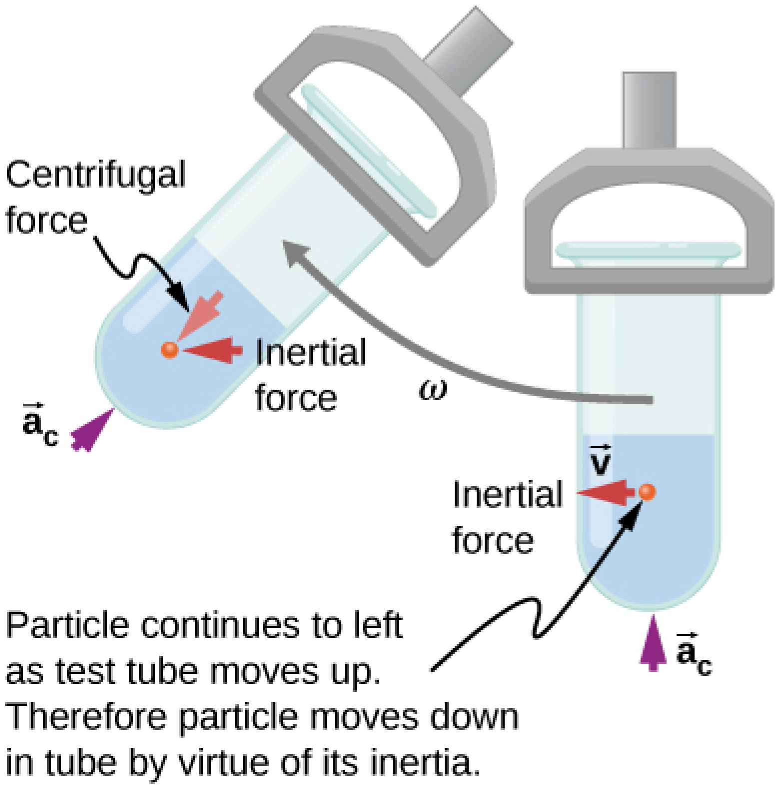
A physicist will choose whatever reference frame is most convenient for the situation being analyzed. There is no problem to a physicist in including inertial forces and Newton’s second law, as usual, if that is more convenient, for example, on a merry-go-round or on a rotating planet. Noninertial (accelerated) frames of reference are used when it is useful to do so. Different frames of reference must be considered in discussing the motion of an astronaut in a spacecraft traveling at speeds near the speed of light, as you will appreciate in the study of the special theory of relativity.

Let us now take a mental ride on a merry-go-round—specifically, a rapidly rotating playground merry-go-round ([link]). You take the merry-go-round to be your frame of reference because you rotate together. When rotating in that noninertial frame of reference, you feel an inertial force that tends to throw you off; this is often referred to as a *centrifugal force* (not to be confused with centripetal force). Centrifugal force is a commonly used term, but it does not actually exist. You must hang on tightly to counteract your inertia (which people often refer to as centrifugal force). In Earth’s frame of reference, there is no force trying to throw you off; we emphasize that centrifugal force is a fiction. You must hang on to make yourself go in a circle because otherwise you would go in a straight line, right off the merry-go-round, in keeping with Newton’s first law. But the force you exert acts toward the center of the circle.



(a) A rider on a merry-go-round feels as if he is being thrown off. This inertial force is sometimes mistakenly called the centrifugal force in an effort to explain the rider's motion in the rotating frame of reference. (b) In an inertial frame of reference and according to Newton's laws, it is his inertia that carries him off (the unshaded rider has $F_{\text{net}} = 0$ and heads in a straight line). A force, $F_{\text{centripetal}}$, is needed to cause a circular path.

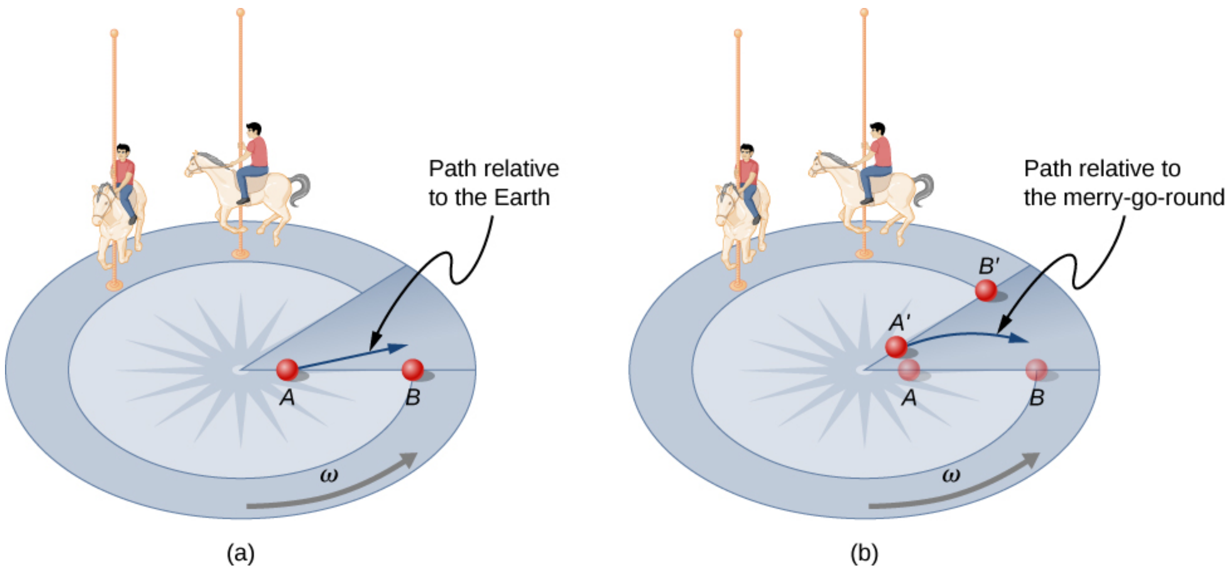
This inertial effect, carrying you away from the center of rotation if there is no centripetal force to cause circular motion, is put to good use in centrifuges ([link]). A centrifuge spins a sample very rapidly, as mentioned earlier in this chapter. Viewed from the rotating frame of reference, the inertial force throws particles outward, hastening their sedimentation. The greater the angular velocity, the greater the centrifugal force. But what really happens is that the inertia of the particles carries them along a line tangent to the circle while the test tube is forced in a circular path by a centripetal force.



Centrifuges use inertia to perform their task. Particles in the fluid sediment settle out because their inertia carries them away from the center of rotation. The large angular velocity of the centrifuge quickens the sedimentation. Ultimately, the particles come into contact with the test tube walls, which then supply the centripetal force needed to make them move in a circle of constant radius.

Let us now consider what happens if something moves in a rotating frame of reference. For example, what if you slide a ball directly away from the center of the merry-go-round, as shown in [link]? The ball follows a straight path relative to Earth (assuming negligible friction) and a path curved to the right on the merry-go-round's surface. A person standing next to the merry-go-round sees the ball moving straight and the merry-go-round rotating

underneath it. In the merry-go-round's frame of reference, we explain the apparent curve to the right by using an inertial force, called the **Coriolis force**, which causes the ball to curve to the right. The Coriolis force can be used by anyone in that frame of reference to explain why objects follow curved paths and allows us to apply Newton's laws in noninertial frames of reference.

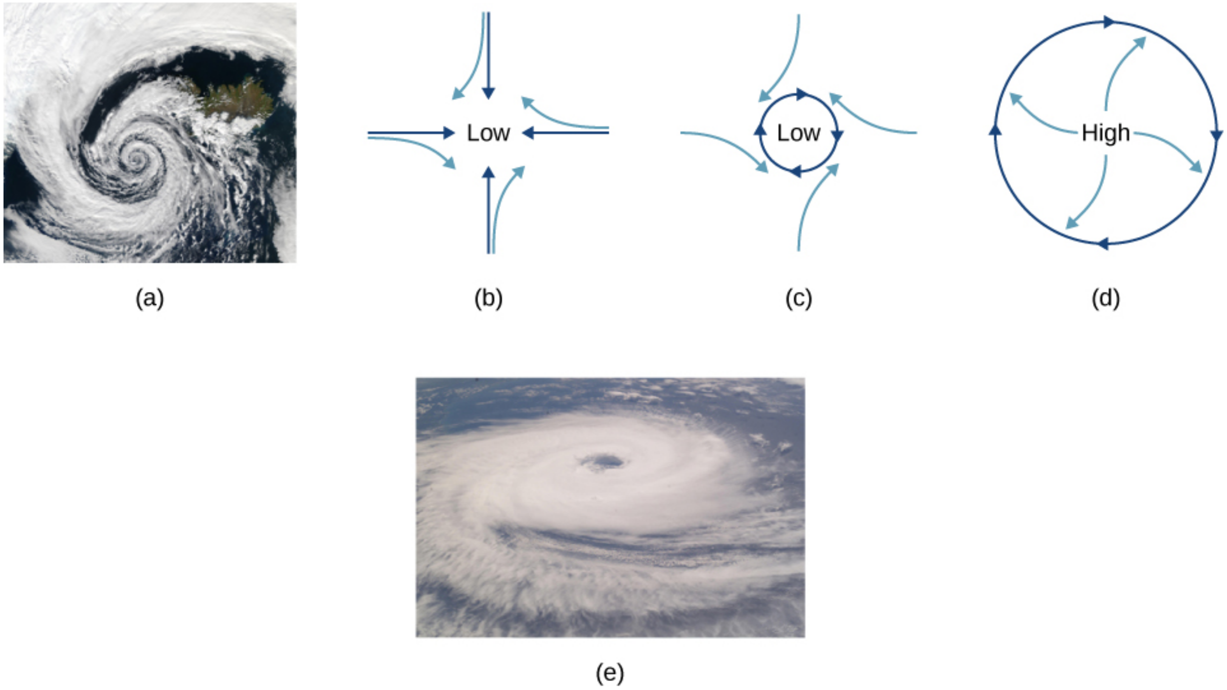


Looking down on the counterclockwise rotation of a merry-go-round, we see that a ball slid straight toward the edge follows a path curved to the right. The person slides the ball toward point *B*, starting at point *A*. Both points rotate to the shaded positions (*A*' and *B*') shown in the time that the ball follows the curved path in the rotating frame and a straight path in Earth's frame.

Up until now, we have considered Earth to be an inertial frame of reference with little or no worry about effects due to its rotation. Yet such effects *do* exist—in the rotation of weather systems, for example. Most consequences of Earth's rotation can be qualitatively understood by analogy with the merry-go-round. Viewed from above the North Pole, Earth rotates counterclockwise, as does the merry-go-round in [link]. As on the merry-

go-round, any motion in Earth’s Northern Hemisphere experiences a Coriolis force to the right. Just the opposite occurs in the Southern Hemisphere; there, the force is to the left. Because Earth’s angular velocity is small, the Coriolis force is usually negligible, but for large-scale motions, such as wind patterns, it has substantial effects.

The Coriolis force causes hurricanes in the Northern Hemisphere to rotate in the counterclockwise direction, whereas tropical cyclones in the Southern Hemisphere rotate in the clockwise direction. (The terms hurricane, typhoon, and tropical storm are regionally specific names for cyclones, which are storm systems characterized by low pressure centers, strong winds, and heavy rains.) [link] helps show how these rotations take place. Air flows toward any region of low pressure, and tropical cyclones contain particularly low pressures. Thus winds flow toward the center of a tropical cyclone or a low-pressure weather system at the surface. In the Northern Hemisphere, these inward winds are deflected to the right, as shown in the figure, producing a counterclockwise circulation at the surface for low-pressure zones of any type. Low pressure at the surface is associated with rising air, which also produces cooling and cloud formation, making low-pressure patterns quite visible from space. Conversely, wind circulation around high-pressure zones is clockwise in the Southern Hemisphere but is less visible because high pressure is associated with sinking air, producing clear skies.



(a) The counterclockwise rotation of this Northern Hemisphere hurricane is a major consequence of the Coriolis force. (b) Without the Coriolis force, air would flow straight into a low-pressure zone, such as that found in tropical cyclones. (c) The Coriolis force deflects the winds to the right, producing a counterclockwise rotation. (d) Wind flowing away from a high-pressure zone is also deflected to the right, producing a clockwise rotation. (e) The opposite direction of rotation is produced by the Coriolis force in the Southern Hemisphere, leading to tropical cyclones. (credit a and credit e: modifications of work by NASA)

The rotation of tropical cyclones and the path of a ball on a merry-go-round can just as well be explained by inertia and the rotation of the system underneath. When noninertial frames are used, inertial forces, such as the Coriolis force, must be invented to explain the curved path. There is no identifiable physical source for these inertial forces. In an inertial frame, inertia explains the path, and no force is found to be without an identifiable source. Either view allows us to describe nature, but a view in an inertial

frame is the simplest in the sense that all forces have origins and explanations.

Summary

- Centripetal force $\vec{\mathbf{F}}_{\mathrm{c}}$ is a “center-seeking” force that always points toward the center of rotation. It is perpendicular to linear velocity and has the magnitude
  Equation:
  $$F_{\mathrm{c}} = ma_{\mathrm{c}}.$$
- Rotating and accelerated frames of reference are noninertial. Inertial forces, such as the Coriolis force, are needed to explain motion in such frames.

Conceptual Questions

Exercise:

Problem:

If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large- or small-diameter tires? Explain.

Exercise:

Problem:

Define centripetal force. Can any type of force (for example, tension, gravitational force, friction, and so on) be a centripetal force? Can any combination of forces be a centripetal force?

Solution:

Centripetal force is defined as any net force causing uniform circular motion. The centripetal force is not a new kind of force. The label “centripetal” refers to *any* force that keeps something turning in a

circle. That force could be tension, gravity, friction, electrical attraction, the normal force, or any other force. Any combination of these could be the source of centripetal force, for example, the centripetal force at the top of the path of a tetherball swung through a vertical circle is the result of both tension and gravity.

Exercise:

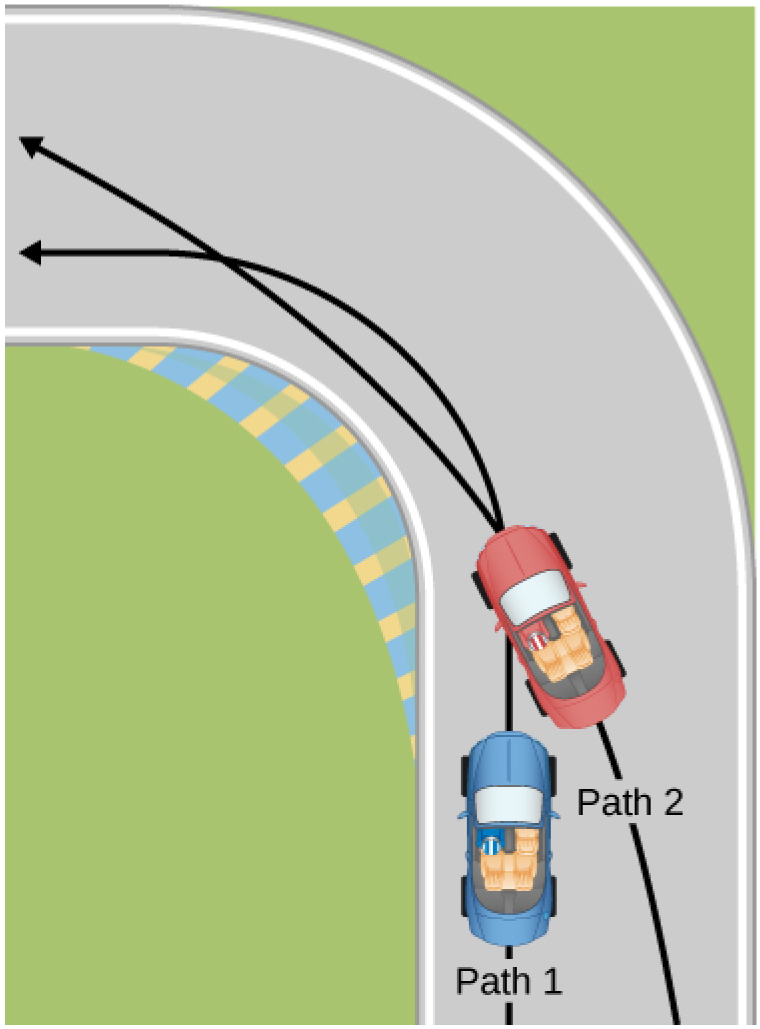
Problem:

If centripetal force is directed toward the center, why do you feel that you are ‘thrown’ away from the center as a car goes around a curve? Explain.

Exercise:

Problem:

Race car drivers routinely cut corners, as shown below (Path 2). Explain how this allows the curve to be taken at the greatest speed.



Solution:

The driver who cuts the corner (on Path 2) has a more gradual curve, with a larger radius. That one will be the better racing line. If the driver goes too fast around a corner using a racing line, he will still slide off the track; the key is to stay at the maximum value of static friction. So, the driver wants maximum possible speed and maximum friction. Consider the equation for centripetal force: $F_{\mathrm{c}} = m\frac{v^2}{r}$ where *v* is speed and *r* is the radius of curvature. So by decreasing the curvature (1/*r*) of the path that the car takes, we reduce the amount of force the tires have to exert on the road, meaning we can now increase the speed, *v*. Looking at this from the point of view of the driver on Path 1, we can reason this way: the sharper the turn, the smaller the turning

circle; the smaller the turning circle, the larger is the required centripetal force. If this centripetal force is not exerted, the result is a skid.

Exercise:

Problem:

Many amusement parks have rides that make vertical loops like the one shown below. For safety, the cars are attached to the rails in such a way that they cannot fall off. If the car goes over the top at just the right speed, gravity alone will supply the centripetal force. What other force acts and what is its direction if:

(a) The car goes over the top at faster than this speed?

(b) The car goes over the top at slower than this speed?

Exercise:

Problem:

What causes water to be removed from clothes in a spin-dryer?

Solution:

The barrel of the dryer provides a centripetal force on the clothes (including the water droplets) to keep them moving in a circular path. As a water droplet comes to one of the holes in the barrel, it will move in a path tangent to the circle.

Exercise:

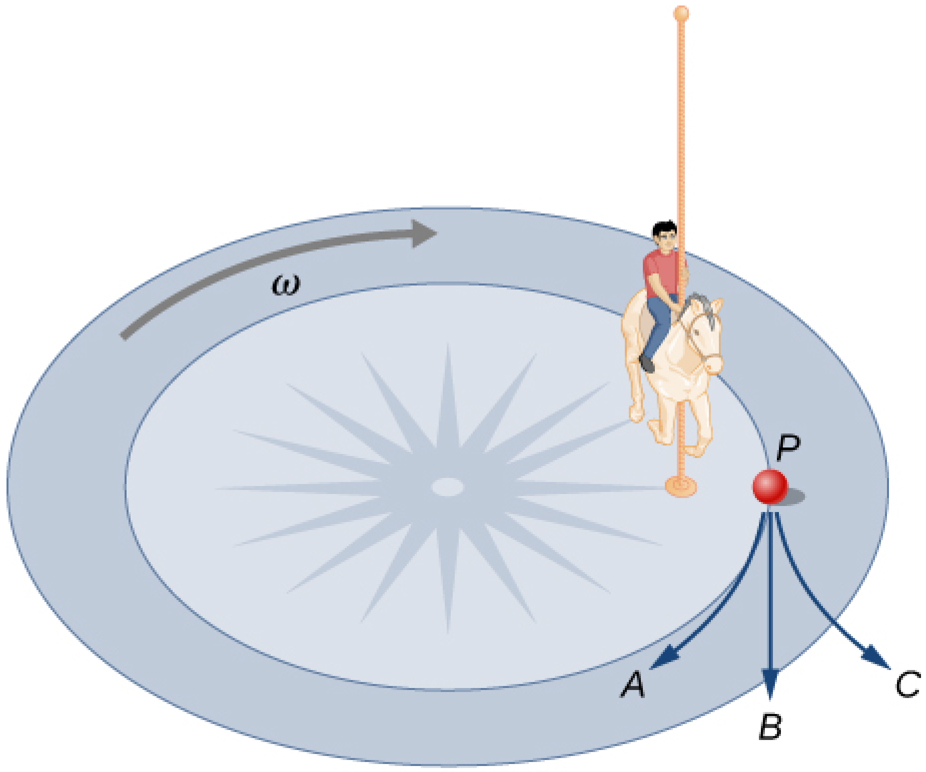
Problem:

As a skater forms a circle, what force is responsible for making his turn? Use a free-body diagram in your answer.

Exercise:

Problem:

Suppose a child is riding on a merry-go-round at a distance about halfway between its center and edge. She has a lunch box resting on wax paper, so that there is very little friction between it and the merry-go-round. Which path shown below will the lunch box take when she lets go? The lunch box leaves a trail in the dust on the merry-go-round. Is that trail straight, curved to the left, or curved to the right? Explain your answer.



Merry-go-round's rotating
frame of reference

Solution:

If there is no friction, then there is no centripetal force. This means that the lunch box will move along a path tangent to the circle, and thus follows path *B*. The dust trail will be straight. This is a result of Newton's first law of motion.

Exercise:

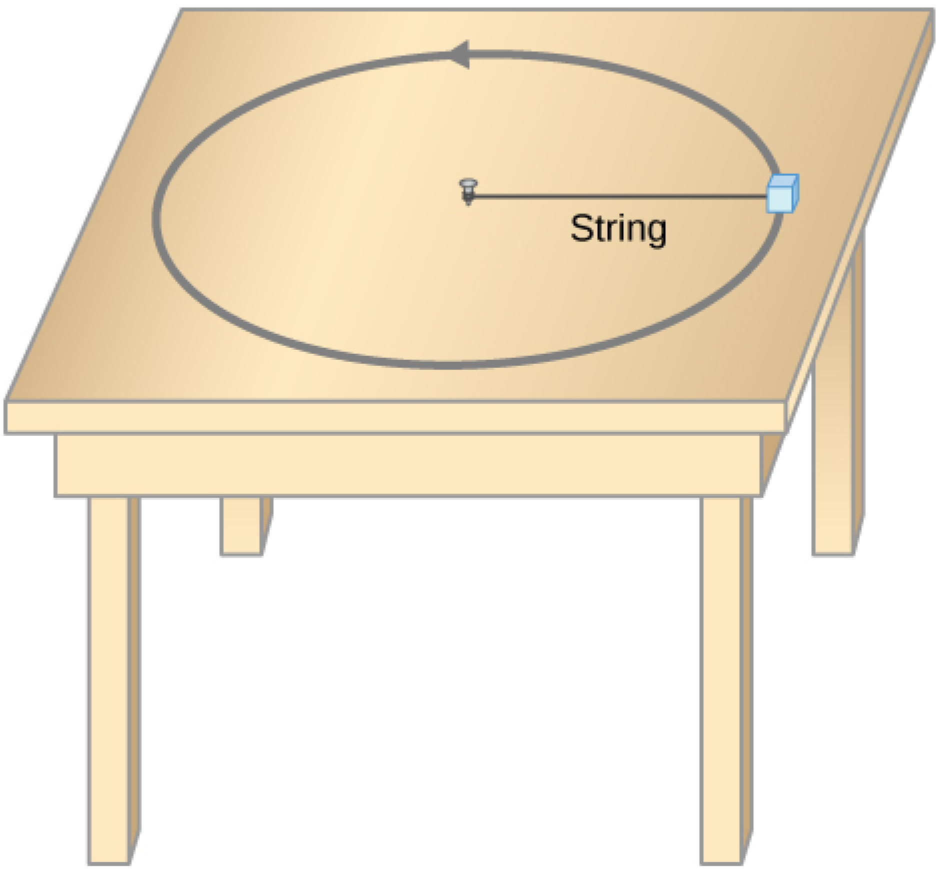
Problem:

Do you feel yourself thrown to either side when you negotiate a curve that is ideally banked for your car's speed? What is the direction of the force exerted on you by the car seat?

Exercise:

Problem:

Suppose a mass is moving in a circular path on a frictionless table as shown below. In Earth's frame of reference, there is no centrifugal force pulling the mass away from the center of rotation, yet there is a force stretching the string attaching the mass to the nail. Using concepts related to centripetal force and Newton's third law, explain what force stretches the string, identifying its physical origin.



Solution:

There must be a centripetal force to maintain the circular motion; this is provided by the nail at the center. Newton's third law explains the phenomenon. The action force is the force of the string on the mass; the reaction force is the force of the mass on the string. This reaction force causes the string to stretch.

Exercise:

Problem:

When a toilet is flushed or a sink is drained, the water (and other material) begins to rotate about the drain on the way down. Assuming no initial rotation and a flow initially directly straight toward the drain, explain what causes the rotation and which direction it has in the Northern Hemisphere. (Note that this is a small effect and in most toilets the rotation is caused by directional water jets.) Would the direction of rotation reverse if water were forced up the drain?

Exercise:

Problem:

A car rounds a curve and encounters a patch of ice with a very low coefficient of kinetic fiction. The car slides off the road. Describe the path of the car as it leaves the road.

Solution:

Since the radial friction with the tires supplies the centripetal force, and friction is nearly 0 when the car encounters the ice, the car will obey Newton’s first law and go off the road in a straight line path, tangent to the curve. A common misconception is that the car will follow a curved path off the road.

Exercise:

Problem:

In one amusement park ride, riders enter a large vertical barrel and stand against the wall on its horizontal floor. The barrel is spun up and the floor drops away. Riders feel as if they are pinned to the wall by a force something like the gravitational force. This is an inertial force sensed and used by the riders to explain events in the rotating frame of reference of the barrel. Explain in an inertial frame of reference (Earth is nearly one) what pins the riders to the wall, and identify all forces acting on them.

Exercise:

Problem:

Two friends are having a conversation. Anna says a satellite in orbit is in free fall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in free fall because the acceleration due to gravity is not $9.80\ \text{m/s}^2$. Who do you agree with and why?

Solution:

Anna is correct. The satellite is freely falling toward Earth due to gravity, even though gravity is weaker at the altitude of the satellite, and g is not $9.80\ \text{m/s}^2$. Free fall does not depend on the value of g; that is, you could experience free fall on Mars if you jumped off Olympus Mons (the tallest volcano in the solar system).

Exercise:

Problem:

A nonrotating frame of reference placed at the center of the Sun is very nearly an inertial one. Why is it not exactly an inertial frame?

Problems

Exercise:

Problem:

(a) A 22.0-kg child is riding a playground merry-go-round that is rotating at 40.0 rev/min. What centripetal force is exerted if he is 1.25 m from its center? (b) What centripetal force is exerted if the merry-go-round rotates at 3.00 rev/min and he is 8.00 m from its center? (c) Compare each force with his weight.

Solution:

a. 483 N; b. 17.4 N; c. 2.24, 0.0807

Exercise:

Problem:

Calculate the centripetal force on the end of a 100-m (radius) wind turbine blade that is rotating at 0.5 rev/s. Assume the mass is 4 kg.

Exercise:

Problem:

What is the ideal banking angle for a gentle turn of 1.20-km radius on a highway with a 105 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?

Solution:

4.14°

Exercise:

Problem:

What is the ideal speed to take a 100.0-m-radius curve banked at a 20.0° angle?

Exercise:

Problem:

(a) What is the radius of a bobsled turn banked at 75.0° and taken at 30.0 m/s, assuming it is ideally banked? (b) Calculate the centripetal acceleration. (c) Does this acceleration seem large to you?

Solution:

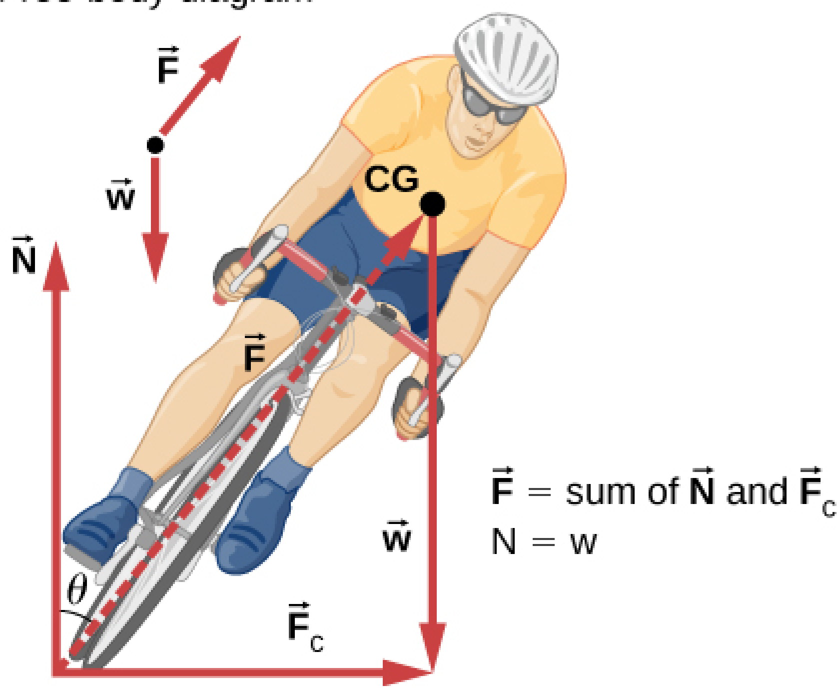
a. 24.6 m; b. $36.6\ \text{m/s}^2$; c. 3.73 times *g*

Exercise:

Problem:

Part of riding a bicycle involves leaning at the correct angle when making a turn, as seen below. To be stable, the force exerted by the ground must be on a line going through the center of gravity. The force on the bicycle wheel can be resolved into two perpendicular components—friction parallel to the road (this must supply the centripetal force) and the vertical normal force (which must equal the system's weight). (a) Show that θ (as defined as shown) is related to the speed *v* and radius of curvature *r* of the turn in the same way as for an ideally banked roadway—that is, $\theta = \tan^{-1}(v^2/rg)$. (b) Calculate θ for a 12.0-m/s turn of radius 30.0 m (as in a race).

Free-body diagram



Exercise:

Problem:

If a car takes a banked curve at less than the ideal speed, friction is needed to keep it from sliding toward the inside of the curve (a problem on icy mountain roads). (a) Calculate the ideal speed to take a 100.0 m radius curve banked at 15.0°. (b) What is the minimum coefficient of friction needed for a frightened driver to take the same curve at 20.0 km/h?

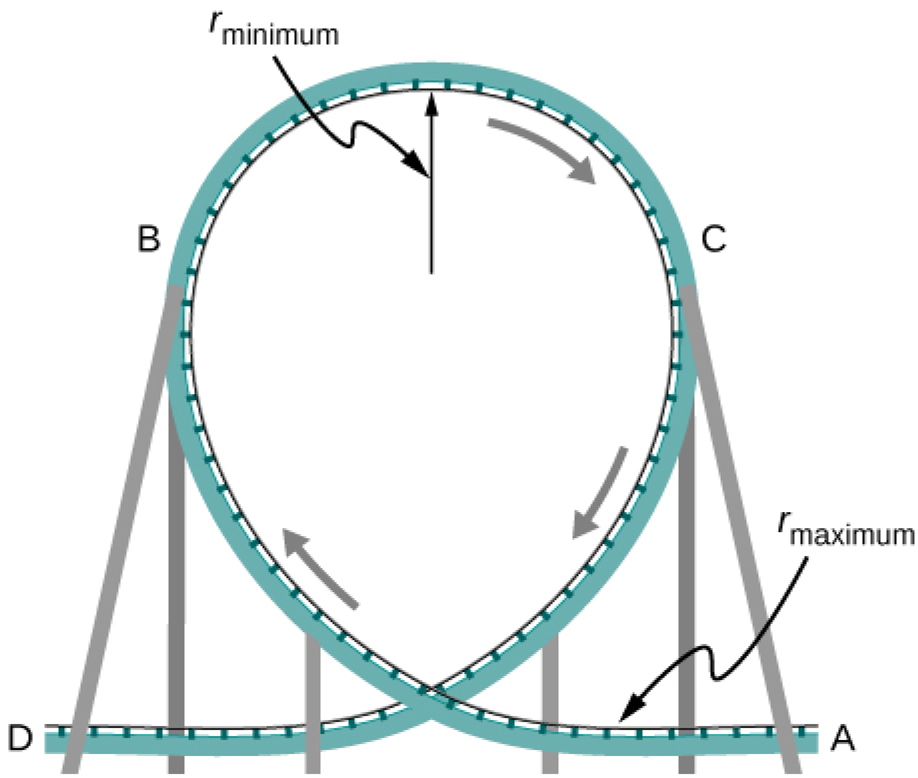
Solution:

a. 16.2 m/s; b. 0.234

Exercise:

Problem:

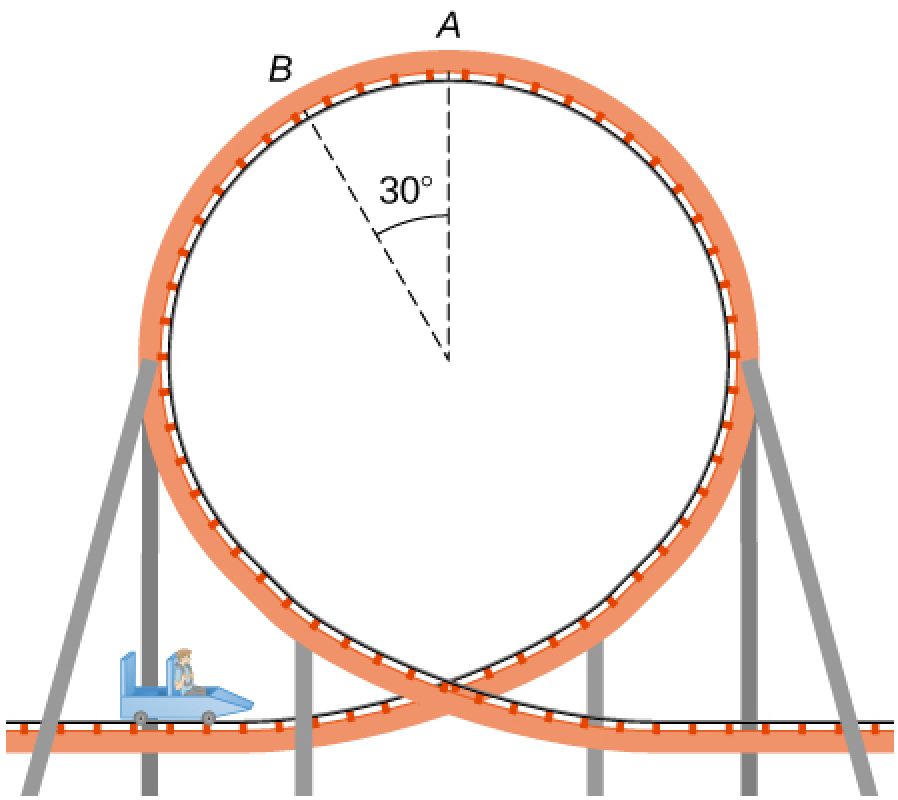
Modern roller coasters have vertical loops like the one shown here. The radius of curvature is smaller at the top than on the sides so that the downward centripetal acceleration at the top will be greater than the acceleration due to gravity, keeping the passengers pressed firmly into their seats. What is the speed of the roller coaster at the top of the loop if the radius of curvature there is 15.0 m and the downward acceleration of the car is 1.50 g?



Exercise:

Problem:

A child of mass 40.0 kg is in a roller coaster car that travels in a loop of radius 7.00 m. At point A the speed of the car is 10.0 m/s, and at point B, the speed is 10.5 m/s. Assume the child is not holding on and does not wear a seat belt. (a) What is the force of the car seat on the child at point A? (b) What is the force of the car seat on the child at point B? (c) What minimum speed is required to keep the child in his seat at point A?



Solution:

a. 179 N; b. 290 N; c. 8.3 m/s

Exercise:

Problem:

In the simple Bohr model of the ground state of the hydrogen atom, the electron travels in a circular orbit around a fixed proton. The radius of the orbit is 5.28×10^{-11} m, and the speed of the electron is 2.18×10^{6} m/s. The mass of an electron is 9.11×10^{-31} kg. What is the force on the electron?

Exercise:

Problem:

Railroad tracks follow a circular curve of radius 500.0 m and are banked at an angle of 5.0°. For trains of what speed are these tracks designed?

Solution:

20.7 m/s

Exercise:

Problem:

The CERN particle accelerator is circular with a circumference of 7.0 km. (a) What is the acceleration of the protons $(m = 1.67 \times 10^{-27}\ \text{kg})$ that move around the accelerator at 5% of the speed of light? (The speed of light is $v = 3.00 \times 10^8\ \text{m/s}$.) (b) What is the force on the protons?

Exercise:

Problem:

A car rounds an unbanked curve of radius 65 m. If the coefficient of static friction between the road and car is 0.70, what is the maximum speed at which the car can traverse the curve without slipping?

Solution:

21 m/s

Exercise:

Problem:

A banked highway is designed for traffic moving at 90.0 km/h. The radius of the curve is 310 m. What is the angle of banking of the highway?

Glossary

banked curve
: curve in a road that is sloping in a manner that helps a vehicle negotiate the curve

centripetal force
: any net force causing uniform circular motion

Coriolis force
: inertial force causing the apparent deflection of moving objects when viewed in a rotating frame of reference

ideal banking
: sloping of a curve in a road, where the angle of the slope allows the vehicle to negotiate the curve at a certain speed without the aid of friction between the tires and the road; the net external force on the vehicle equals the horizontal centripetal force in the absence of friction

inertial force
: force that has no physical origin

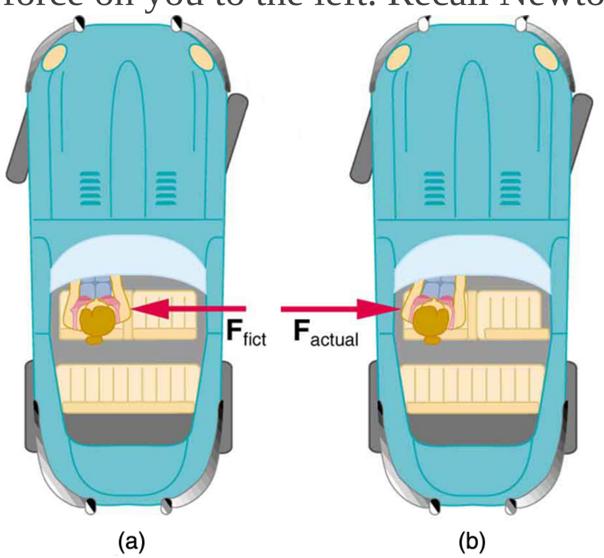
noninertial frame of reference
: accelerated frame of reference

Fictitious Forces and Non-inertial Frames: The Coriolis Force

- Discuss the inertial frame of reference.
- Discuss the non-inertial frame of reference.
- Describe the effects of the Coriolis force.

What do taking off in a jet airplane, turning a corner in a car, riding a merry-go-round, and the circular motion of a tropical cyclone have in common? Each exhibits fictitious forces—unreal forces that arise from motion and may *seem* real, because the observer's frame of reference is accelerating or rotating.

When taking off in a jet, most people would agree it feels as if you are being pushed back into the seat as the airplane accelerates down the runway. Yet a physicist would say that *you* tend to remain stationary while the *seat* pushes forward on you, and there is no real force backward on you. An even more common experience occurs when you make a tight curve in your car—say, to the right. You feel as if you are thrown (that is, *forced*) toward the left relative to the car. Again, a physicist would say that *you* are going in a straight line but the *car* moves to the right, and there is no real force on you to the left. Recall Newton's first law.

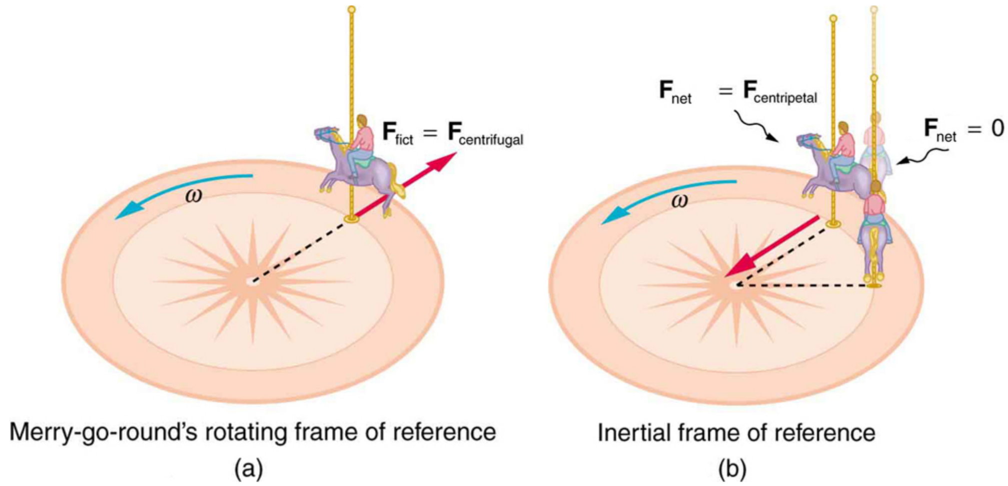


(a) The car driver feels herself forced to the left relative to the car when she makes a right turn. This is a fictitious force arising

from the use of the car as a frame of reference. (b) In the Earth's frame of reference, the driver moves in a straight line, obeying Newton's first law, and the car moves to the right. There is no real force to the left on the driver relative to Earth. There is a real force to the right on the car to make it turn.

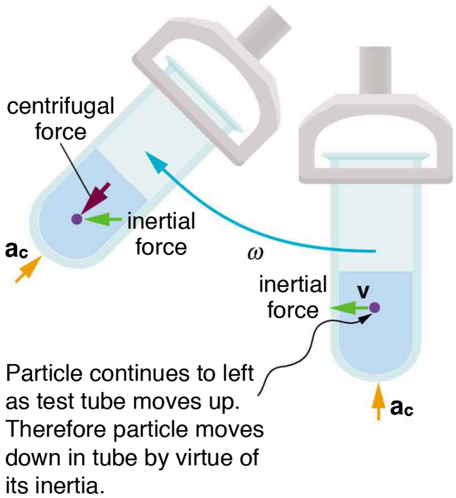
We can reconcile these points of view by examining the frames of reference used. Let us concentrate on people in a car. Passengers instinctively use the car as a frame of reference, while a physicist uses Earth. The physicist chooses Earth because it is very nearly an inertial frame of reference—one in which all forces are real (that is, in which all forces have an identifiable physical origin). In such a frame of reference, Newton's laws of motion take the form given in Dynamics: Newton's Laws of Motion The car is a **non-inertial frame of reference** because it is accelerated to the side. The force to the left sensed by car passengers is a **fictitious force** having no physical origin. There is nothing real pushing them left—the car, as well as the driver, is actually accelerating to the right.

Let us now take a mental ride on a merry-go-round—specifically, a rapidly rotating playground merry-go-round. You take the merry-go-round to be your frame of reference because you rotate together. In that non-inertial frame, you feel a fictitious force, named **centrifugal force (**not to be confused with centripetal force**)**, trying to throw you off. You must hang on tightly to counteract the centrifugal force. In Earth's frame of reference, there is no force trying to throw you off. Rather you must hang on to make yourself go in a circle because otherwise you would go in a straight line, right off the merry-go-round.



(a) A rider on a merry-go-round feels as if he is being thrown off. This fictitious force is called the centrifugal force—it explains the rider's motion in the rotating frame of reference. (b) In an inertial frame of reference and according to Newton's laws, it is his inertia that carries him off and not a real force (the unshaded rider has $F_{\text{net}} = 0$ and heads in a straight line). A real force, $F_{\text{centripetal}}$, is needed to cause a circular path.

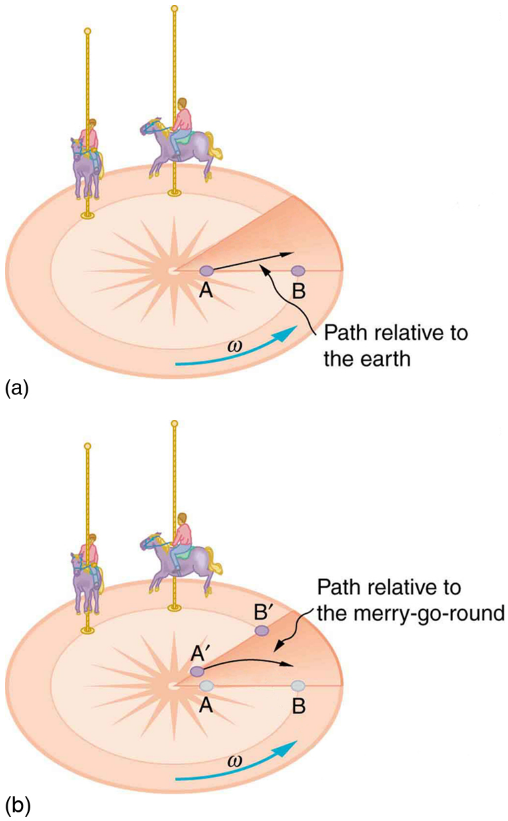
This inertial effect, carrying you away from the center of rotation if there is no centripetal force to cause circular motion, is put to good use in centrifuges (see [link]). A centrifuge spins a sample very rapidly, as mentioned earlier in this chapter. Viewed from the rotating frame of reference, the fictitious centrifugal force throws particles outward, hastening their sedimentation. The greater the angular velocity, the greater the centrifugal force. But what really happens is that the inertia of the particles carries them along a line tangent to the circle while the test tube is forced in a circular path by a centripetal force.



Centrifuges use inertia to perform their task. Particles in the fluid sediment come out because their inertia carries them away from the center of rotation. The large angular velocity of the centrifuge quickens the sedimentation. Ultimately, the particles will come into contact with the test tube walls, which will then supply the centripetal force needed to make them move in a circle of constant radius.

Let us now consider what happens if something moves in a frame of reference that rotates. For example, what if you slide a ball directly away

from the center of the merry-go-round, as shown in [link]? The ball follows a straight path relative to Earth (assuming negligible friction) and a path curved to the right on the merry-go-round's surface. A person standing next to the merry-go-round sees the ball moving straight and the merry-go-round rotating underneath it. In the merry-go-round's frame of reference, we explain the apparent curve to the right by using a fictitious force, called the **Coriolis force**, that causes the ball to curve to the right. The fictitious Coriolis force can be used by anyone in that frame of reference to explain why objects follow curved paths and allows us to apply Newton's Laws in non-inertial frames of reference.



Looking down on the counterclockwise rotation of a merry-go-round, we see that a ball slid straight toward the edge follows a path curved to the right. The person slides the ball toward point B, starting at

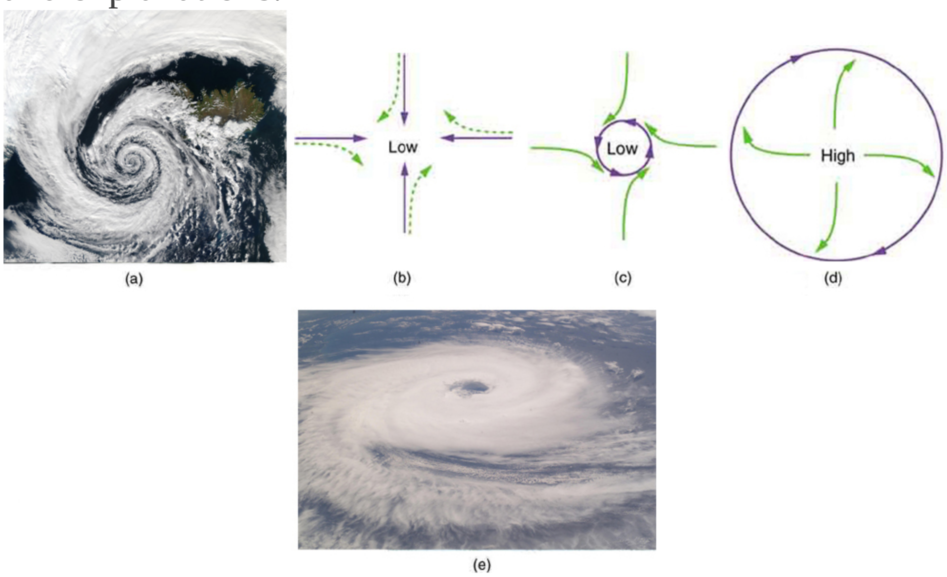
point A. Both points rotate to the shaded positions (A’ and B’) shown in the time that the ball follows the curved path in the rotating frame and a straight path in Earth’s frame.

Up until now, we have considered Earth to be an inertial frame of reference with little or no worry about effects due to its rotation. Yet such effects *do* exist—in the rotation of weather systems, for example. Most consequences of Earth’s rotation can be qualitatively understood by analogy with the merry-go-round. Viewed from above the North Pole, Earth rotates counterclockwise, as does the merry-go-round in [link]. As on the merry-go-round, any motion in Earth’s northern hemisphere experiences a Coriolis force to the right. Just the opposite occurs in the southern hemisphere; there, the force is to the left. Because Earth’s angular velocity is small, the Coriolis force is usually negligible, but for large-scale motions, such as wind patterns, it has substantial effects.

The Coriolis force causes hurricanes in the northern hemisphere to rotate in the counterclockwise direction, while the tropical cyclones (what hurricanes are called below the equator) in the southern hemisphere rotate in the clockwise direction. The terms hurricane, typhoon, and tropical storm are regionally-specific names for tropical cyclones, storm systems characterized by low pressure centers, strong winds, and heavy rains. [link] helps show how these rotations take place. Air flows toward any region of low pressure, and tropical cyclones contain particularly low pressures. Thus winds flow toward the center of a tropical cyclone or a low-pressure weather system at the surface. In the northern hemisphere, these inward winds are deflected to the right, as shown in the figure, producing a counterclockwise circulation at the surface for low-pressure zones of any type. Low pressure at the surface is associated with rising air, which also produces cooling and cloud formation, making low-pressure patterns quite

visible from space. Conversely, wind circulation around high-pressure zones is clockwise in the northern hemisphere but is less visible because high pressure is associated with sinking air, producing clear skies.

The rotation of tropical cyclones and the path of a ball on a merry-go-round can just as well be explained by inertia and the rotation of the system underneath. When non-inertial frames are used, fictitious forces, such as the Coriolis force, must be invented to explain the curved path. There is no identifiable physical source for these fictitious forces. In an inertial frame, inertia explains the path, and no force is found to be without an identifiable source. Either view allows us to describe nature, but a view in an inertial frame is the simplest and truest, in the sense that all forces have real origins and explanations.



(a) The counterclockwise rotation of this northern hemisphere hurricane is a major consequence of the Coriolis force. (credit: NASA) (b) Without the Coriolis force, air would flow straight into a low-pressure zone, such as that found in tropical cyclones. (c) The Coriolis force deflects the winds to the right, producing a counterclockwise rotation. (d) Wind flowing away from a high-pressure zone is also deflected to the right, producing a clockwise rotation. (e) The opposite direction of rotation is produced by

the Coriolis force in the southern hemisphere, leading to tropical cyclones. (credit: NASA)

Section Summary

- Rotating and accelerated frames of reference are non-inertial.
- Fictitious forces, such as the Coriolis force, are needed to explain motion in such frames.

Conceptual Questions

Exercise:

Problem:

When a toilet is flushed or a sink is drained, the water (and other material) begins to rotate about the drain on the way down. Assuming no initial rotation and a flow initially directly straight toward the drain, explain what causes the rotation and which direction it has in the northern hemisphere. (Note that this is a small effect and in most toilets the rotation is caused by directional water jets.) Would the direction of rotation reverse if water were forced up the drain?

Exercise:

Problem:

Is there a real force that throws water from clothes during the spin cycle of a washing machine? Explain how the water is removed.

Exercise:

Problem:

In one amusement park ride, riders enter a large vertical barrel and stand against the wall on its horizontal floor. The barrel is spun up and the floor drops away. Riders feel as if they are pinned to the wall by a force something like the gravitational force. This is a fictitious force sensed and used by the riders to explain events in the rotating frame of reference of the barrel. Explain in an inertial frame of reference (Earth is nearly one) what pins the riders to the wall, and identify all of the real forces acting on them.

Exercise:

Problem:

Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?

Exercise:

Problem:

Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not $9.80\ \text{m/s}^2$. Who do you agree with and why?

Exercise:

Problem:

A non-rotating frame of reference placed at the center of the Sun is very nearly an inertial one. Why is it not exactly an inertial frame?

Glossary

fictitious force
: a force having no physical origin

centrifugal force
: a fictitious force that tends to throw an object off when the object is rotating in a non-inertial frame of reference

Coriolis force
: the fictitious force causing the apparent deflection of moving objects when viewed in a rotating frame of reference

non-inertial frame of reference
: an accelerated frame of reference

Newton’s Universal Law of Gravitation

- Explain Earth’s gravitational force.
- Describe the gravitational effect of the Moon on Earth.
- Discuss weightlessness in space.
- Examine the Cavendish experiment

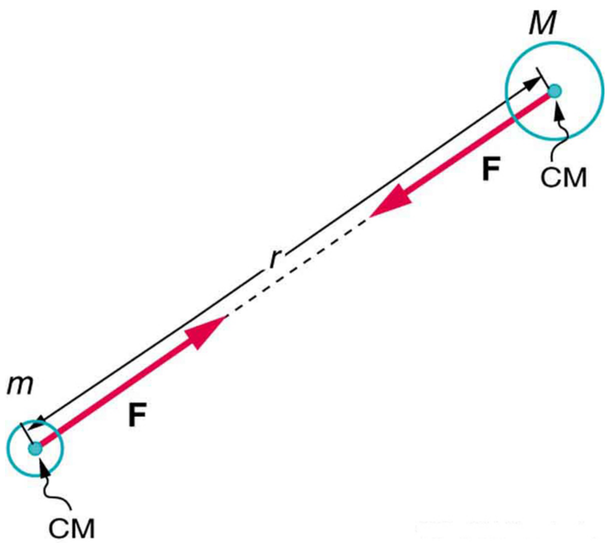
What do aching feet, a falling apple, and the orbit of the Moon have in common? Each is caused by the gravitational force. Our feet are strained by supporting our weight—the force of Earth’s gravity on us. An apple falls from a tree because of the same force acting a few meters above Earth’s surface. And the Moon orbits Earth because gravity is able to supply the necessary centripetal force at a distance of hundreds of millions of meters. In fact, the same force causes planets to orbit the Sun, stars to orbit the center of the galaxy, and galaxies to cluster together. Gravity is another example of underlying simplicity in nature. It is the weakest of the four basic forces found in nature, and in some ways the least understood. It is a force that acts at a distance, without physical contact, and is expressed by a formula that is valid everywhere in the universe, for masses and distances that vary from the tiny to the immense.

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See [link]. But Newton was not the first to suspect that the same force caused both our weight and the motion of planets. His forerunner Galileo Galilei had contended that falling bodies and planetary motions had the same cause. Some of Newton’s contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph—it had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose a mechanism that caused them to follow these paths and not others.

According to early accounts, Newton was inspired to make the connection between falling bodies and astronomical motions when he saw an apple fall from a tree and realized that if the gravitational force could extend above the ground to a tree, it might also reach the Sun. The inspiration of Newton's apple is a part of worldwide folklore and may even be based in fact. Great importance is attached to it because Newton's

universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Scientists still expect underlying simplicity to emerge from their ongoing inquiries into nature.

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Stated in modern language, **Newton's universal law of gravitation** states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.



Gravitational attraction is along a line joining the centers of mass of these two bodies. The magnitude of the force is the same on each, consistent with Newton’s third law.

Note:
Misconception Alert
The magnitude of the force on each object (one has larger mass than the other) is the same, consistent with Newton’s third law.

The bodies we are dealing with tend to be large. To simplify the situation we assume that the body acts as if its entire mass is concentrated at one specific point called the **center of mass** (CM), which will be further explored in Linear Momentum and Collisions. For two bodies having masses m and M with a distance r between their centers of mass, the equation for Newton’s universal law of gravitation is
Equation:

$$F = G\frac{\mathrm{mM}}{r^2},$$

where F is the magnitude of the gravitational force and G is a proportionality factor called the **gravitational constant**. G is a universal gravitational constant—that is, it is thought to be the same everywhere in the universe. It has been measured experimentally to be
Equation:

$$G = 6.674 \times 10^{-11}\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$$

in SI units. Note that the units of G are such that a force in newtons is obtained from $F = G\frac{\text{mM}}{r^2}$, when considering masses in kilograms and distance in meters. For example, two 1.000 kg masses separated by 1.000 m will experience a gravitational attraction of 6.674×10^{-11} N. This is an extraordinarily small force. The small magnitude of the gravitational force is consistent with everyday experience. We are unaware that even large objects like mountains exert gravitational forces on us. In fact, our body weight is the force of attraction of the *entire Earth* on us with a mass of 6×10^{24} kg.

Recall that the acceleration due to gravity g is about 9.80 m/s^2 on Earth. We can now determine why this is so. The weight of an object *mg* is the gravitational force between it and Earth. Substituting *mg* for F in Newton's universal law of gravitation gives

Equation:

$$\text{mg} = G\frac{\text{mM}}{r^2},$$

where m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See [link]. The mass m of the object cancels, leaving an equation for g:

Equation:

$$g = G\frac{M}{r^2}.$$

Substituting known values for Earth's mass and radius (to three significant figures),

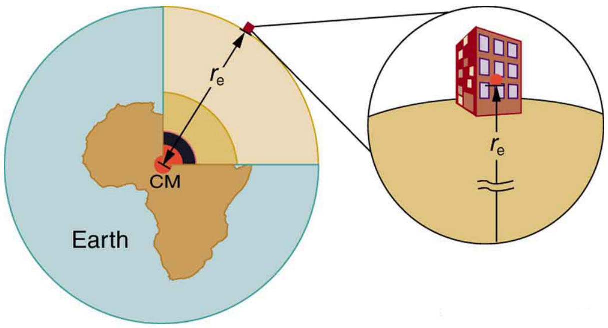
Equation:

$$g = \left(6.67 \times 10^{-11}\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \times \frac{5.98 \times 10^{24}\text{ kg}}{(6.38 \times 10^6\text{ m})^2},$$

and we obtain a value for the acceleration of a falling body:

Equation:

$$g = 9.80 \text{ m/s}^2.$$



The distance between the centers of mass of Earth and an object on its surface is very nearly the same as the radius of Earth, because Earth is so much larger than the object.

This is the expected value *and is independent of the body's mass*. Newton's law of gravitation takes Galileo's observation that all masses fall with the same acceleration a step further, explaining the observation in terms of a force that causes objects to fall—in fact, in terms of a universally existing force of attraction between masses.

Note:

Take-Home Experiment

Take a marble, a ball, and a spoon and drop them from the same height. Do they hit the floor at the same time? If you drop a piece of paper as well, does it behave like the other objects? Explain your observations.

Note:
Making Connections
Attempts are still being made to understand the gravitational force. As we shall see in Particle Physics, modern physics is exploring the connections of gravity to other forces, space, and time. General relativity alters our view of gravitation, leading us to think of gravitation as bending space and time.

In the following example, we make a comparison similar to one made by Newton himself. He noted that if the gravitational force caused the Moon to orbit Earth, then the acceleration due to gravity should equal the centripetal acceleration of the Moon in its orbit. Newton found that the two accelerations agreed "pretty nearly."

Example:
Earth's Gravitational Force Is the Centripetal Force Making the Moon Move in a Curved Path
(a) Find the acceleration due to Earth's gravity at the distance of the Moon.
(b) Calculate the centripetal acceleration needed to keep the Moon in its orbit (assuming a circular orbit about a fixed Earth), and compare it with the value of the acceleration due to Earth's gravity that you have just found.
Strategy for (a)
This calculation is the same as the one finding the acceleration due to gravity at Earth's surface, except that $r$is the distance from the center of Earth to the center of the Moon. The radius of the Moon's nearly circular orbit is 3.84×10^{8} m.
Solution for (a)
Substituting known values into the expression for g found above, remembering that M is the mass of Earth not the Moon, yields
Equation:

$$\begin{aligned} g &= G\frac{M}{r^2} = \left(6.67 \times 10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) \times \frac{5.98\times10^{24}\ \text{kg}}{(3.84\times10^{8}\ \text{m})^2} \\ &= 2.70 \times 10^{-3}\ \text{m/s.}^2 \end{aligned}$$

Strategy for (b)

Centripetal acceleration can be calculated using either form of

Equation:

$$\left.\begin{aligned} a_c = \frac{v^2}{r} \\ a_c = r\omega^2 \end{aligned}\right\}.$$

We choose to use the second form:

Equation:

$$a_c = r\omega^2,$$

where ω is the angular velocity of the Moon about Earth.

Solution for (b)

Given that the period (the time it takes to make one complete rotation) of the Moon's orbit is 27.3 days, (d) and using

Equation:

$$1\ \text{d} \times 24\frac{\text{hr}}{\text{d}} \times 60\frac{\text{min}}{\text{hr}} \times 60\frac{\text{s}}{\text{min}} = 86{,}400\ \text{s}$$

we see that

Equation:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\ \text{rad}}{(27.3\ \text{d})(86{,}400\ \text{s/d})} = 2.66 \times 10^{-6}\,\frac{\text{rad}}{\text{s}}.$$

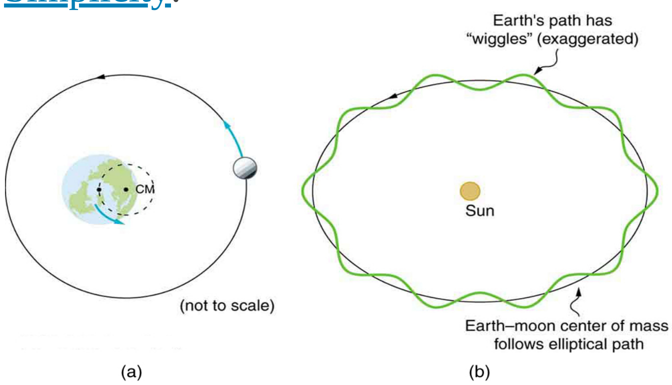
The centripetal acceleration is

Equation:

$$\begin{aligned} a_c &= r\omega^2 = (3.84 \times 10^{8}\ \text{m})(2.66 \times 10^{-6}\ \text{rad/s})^2 \\ &= 2.72 \times 10^{-3}\ \text{m/s.}^2 \end{aligned}$$

The direction of the acceleration is toward the center of the Earth.
Discussion
The centripetal acceleration of the Moon found in (b) differs by less than 1% from the acceleration due to Earth's gravity found in (a). This agreement is approximate because the Moon's orbit is slightly elliptical, and Earth is not stationary (rather the Earth-Moon system rotates about its center of mass, which is located some 1700 km below Earth's surface). The clear implication is that Earth's gravitational force causes the Moon to orbit Earth.

Why does Earth not remain stationary as the Moon orbits it? This is because, as expected from Newton's third law, if Earth exerts a force on the Moon, then the Moon should exert an equal and opposite force on Earth (see [link]). We do not sense the Moon's effect on Earth's motion, because the Moon's gravity moves our bodies right along with Earth but there are other signs on Earth that clearly show the effect of the Moon's gravitational force as discussed in Satellites and Kepler's Laws: An Argument for Simplicity.

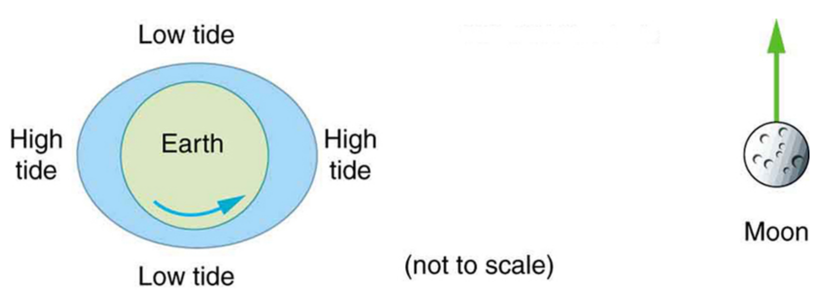


(a) Earth and the Moon rotate approximately once a month around their common center of mass. (b) Their center of mass orbits the Sun in an elliptical orbit, but Earth's path around the Sun has "wiggles" in it. Similar wiggles in the paths of stars have been observed and are

considered direct evidence of planets orbiting those stars. This is important because the planets' reflected light is often too dim to be observed.

Tides

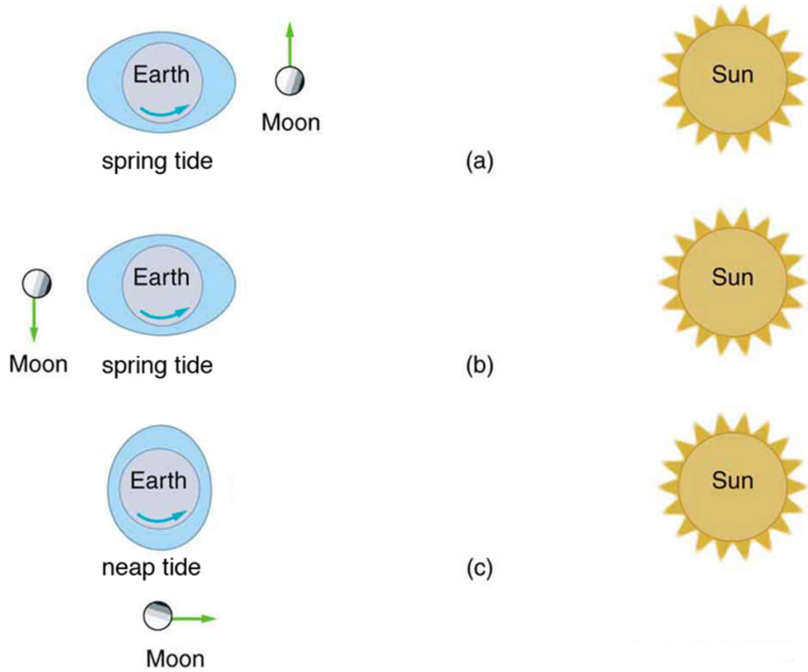
Ocean tides are one very observable result of the Moon's gravity acting on Earth. [link] is a simplified drawing of the Moon's position relative to the tides. Because water easily flows on Earth's surface, a high tide is created on the side of Earth nearest to the Moon, where the Moon's gravitational pull is strongest. Why is there also a high tide on the opposite side of Earth? The answer is that Earth is pulled toward the Moon more than the water on the far side, because Earth is closer to the Moon. So the water on the side of Earth closest to the Moon is pulled away from Earth, and Earth is pulled away from water on the far side. As Earth rotates, the tidal bulge (an effect of the tidal forces between an orbiting natural satellite and the primary planet that it orbits) keeps its orientation with the Moon. Thus there are two tides per day (the actual tidal period is about 12 hours and 25.2 minutes), because the Moon moves in its orbit each day as well).



The Moon causes ocean tides by attracting the water on the near side more than Earth, and by attracting Earth more than the water on the far side. The distances and sizes are

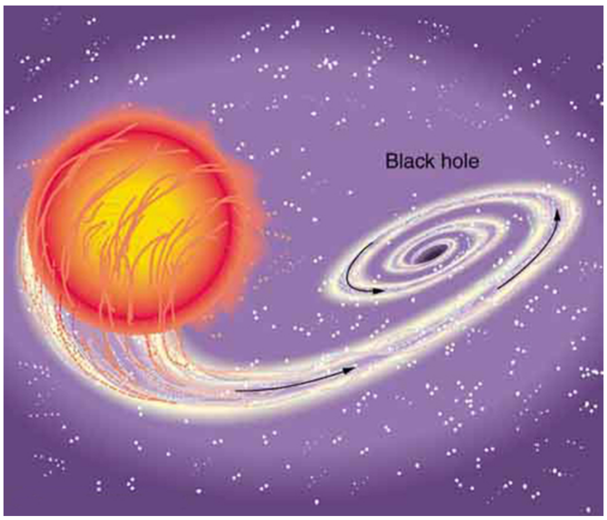
not to scale. For this simplified representation of the Earth-Moon system, there are two high and two low tides per day at any location, because Earth rotates under the tidal bulge.

The Sun also affects tides, although it has about half the effect of the Moon. However, the largest tides, called spring tides, occur when Earth, the Moon, and the Sun are aligned. The smallest tides, called neap tides, occur when the Sun is at a 90º angle to the Earth-Moon alignment.



(a, b) Spring tides: The highest tides occur when Earth, the Moon, and the Sun are aligned. (c) Neap tide: The lowest tides occur when the Sun lies at 90º to the Earth-Moon alignment. Note that this figure is not drawn to scale.

Tides are not unique to Earth but occur in many astronomical systems. The most extreme tides occur where the gravitational force is the strongest and varies most rapidly, such as near black holes (see [link]). A few likely candidates for black holes have been observed in our galaxy. These have masses greater than the Sun but have diameters only a few kilometers across. The tidal forces near them are so great that they can actually tear matter from a companion star.



A black hole is an object with such strong gravity that not even light can escape it. This black hole was created by the supernova of one star in a two-star system. The tidal forces created by the black hole are so great that it tears matter from the companion star. This matter is compressed and heated as it is sucked into the black hole, creating light and X-rays observable from Earth.

”Weightlessness” and Microgravity

In contrast to the tremendous gravitational force near black holes is the apparent gravitational field experienced by astronauts orbiting Earth. What is the effect of “weightlessness” upon an astronaut who is in orbit for months? Or what about the effect of weightlessness upon plant growth? Weightlessness doesn’t mean that an astronaut is not being acted upon by the gravitational force. There is no “zero gravity” in an astronaut’s orbit. The term just means that the astronaut is in free-fall, accelerating with the acceleration due to gravity. If an elevator cable breaks, the passengers inside will be in free fall and will experience weightlessness. You can experience short periods of weightlessness in some rides in amusement parks.

Astronauts experiencing weightlessness on board the International Space Station. (credit: NASA)

Microgravity refers to an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface. Many interesting biology and physics topics have been studied over the past three decades in the presence of microgravity. Of immediate concern is the effect on astronauts of extended times in outer space, such as at the International Space Station. Researchers have observed that muscles will atrophy (waste away) in this environment. There is also a corresponding loss of bone mass. Study continues on cardiovascular adaptation to space flight. On Earth, blood pressure is usually higher in the feet than in the head, because the higher column of blood exerts a downward force on it, due to gravity. When standing, 70% of your blood is below the level of the heart, while in a horizontal position, just the opposite occurs. What difference does the absence of this pressure differential have upon the heart?

Some findings in human physiology in space can be clinically important to the management of diseases back on Earth. On a somewhat negative note, spaceflight is known to affect the human immune system, possibly making the crew members more vulnerable to infectious diseases. Experiments flown in space also have shown that some bacteria grow faster in microgravity than they do on Earth. However, on a positive note, studies indicate that microbial antibiotic production can increase by a factor of two in space-grown cultures. One hopes to be able to understand these mechanisms so that similar successes can be achieved on the ground. In another area of physics space research, inorganic crystals and protein crystals have been grown in outer space that have much higher quality than any grown on Earth, so crystallography studies on their structure can yield much better results.

Plants have evolved with the stimulus of gravity and with gravity sensors. Roots grow downward and shoots grow upward. Plants might be able to provide a life support system for long duration space missions by regenerating the atmosphere, purifying water, and producing food. Some studies have indicated that plant growth and development are not affected by gravity, but there is still uncertainty about structural changes in plants grown in a microgravity environment.

The Cavendish Experiment: Then and Now

As previously noted, the universal gravitational constant G is determined experimentally. This definition was first done accurately by Henry Cavendish (1731–1810), an English scientist, in 1798, more than 100 years after Newton published his universal law of gravitation. The measurement of G is very basic and important because it determines the strength of one of the four forces in nature. Cavendish's experiment was very difficult because he measured the tiny gravitational attraction between two ordinary-sized masses (tens of kilograms at most), using apparatus like that in [link]. Remarkably, his value for G differs by less than 1% from the best modern value.

One important consequence of knowing G was that an accurate value for Earth's mass could finally be obtained. This was done by measuring the acceleration due to gravity as accurately as possible and then calculating the mass of Earth M from the relationship Newton's universal law of gravitation gives

Equation:

$$\mathrm{mg} = G\frac{\mathrm{mM}}{r^2},$$

where m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See [link]. The mass m of the object cancels, leaving an equation for g:

Equation:

$$g = G\frac{M}{r^2}.$$

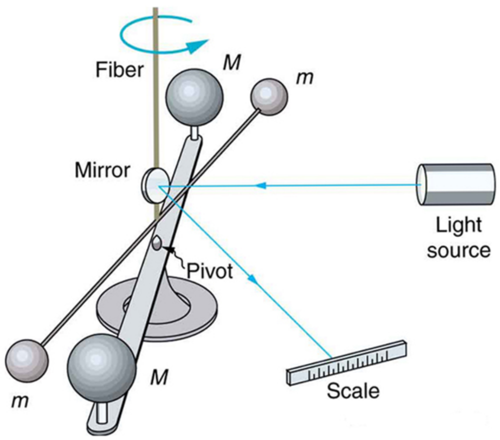
Rearranging to solve for M yields

Equation:

$$M=\frac{gr^2}{G}.$$

So M can be calculated because all quantities on the right, including the radius of Earth r, are known from direct measurements. We shall see in Satellites and Kepler's Laws: An Argument for Simplicity that knowing G also allows for the determination of astronomical masses. Interestingly, of all the fundamental constants in physics, G is by far the least well determined.

The Cavendish experiment is also used to explore other aspects of gravity. One of the most interesting questions is whether the gravitational force depends on substance as well as mass—for example, whether one kilogram of lead exerts the same gravitational pull as one kilogram of water. A Hungarian scientist named Roland von Eötvös pioneered this inquiry early in the 20th century. He found, with an accuracy of five parts per billion, that the gravitational force does not depend on the substance. Such experiments continue today, and have improved upon Eötvös' measurements. Cavendish-type experiments such as those of Eric Adelberger and others at the University of Washington, have also put severe limits on the possibility of a fifth force and have verified a major prediction of general relativity—that gravitational energy contributes to rest mass. Ongoing measurements there use a torsion balance and a parallel plate (not spheres, as Cavendish used) to examine how Newton's law of gravitation works over sub-millimeter distances. On this small-scale, do gravitational effects depart from the inverse square law? So far, no deviation has been observed.



Cavendish used an apparatus like this to measure the gravitational attraction between the two suspended spheres (m) and the two on the stand (M) by observing the amount of torsion (twisting) created in the fiber. Distance between the masses can be varied to check the dependence of the force on distance. Modern experiments of this type continue to explore gravity.

Section Summary

- Newton's universal law of gravitation: Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In equation form, this is

Equation:

$$F = G\frac{\mathrm{mM}}{r^2},$$

where F is the magnitude of the gravitational force. G is the gravitational constant, given by $G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.

- Newton's law of gravitation applies universally.

Conceptual Questions

Exercise:

Problem:

Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?

Exercise:

Problem:

Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not $9.80\ \mathrm{m/s^2}$. Who do you agree with and why?

Exercise:

Problem:

Draw a free body diagram for a satellite in an elliptical orbit showing why its speed increases as it approaches its parent body and decreases as it moves away.

Exercise:

Problem:

Newton’s laws of motion and gravity were among the first to convincingly demonstrate the underlying simplicity and unity in nature. Many other examples have since been discovered, and we now expect to find such underlying order in complex situations. Is there proof that such order will always be found in new explorations?

Problem Exercises

Exercise:

Problem:

(a) Calculate Earth’s mass given the acceleration due to gravity at the North Pole is 9.830 m/s^2 and the radius of the Earth is 6371 km from center to pole.

(b) Compare this with the accepted value of $5.979 \times 10^{24} \text{ kg}$.

Solution:

a) $5.979 \times 10^{24} \text{ kg}$

b) This is identical to the best value to three significant figures.

Exercise:

Problem:

(a) Calculate the magnitude of the acceleration due to gravity on the surface of Earth due to the Moon.

(b) Calculate the magnitude of the acceleration due to gravity at Earth due to the Sun.

(c) Take the ratio of the Moon’s acceleration to the Sun’s and comment on why the tides are predominantly due to the Moon in spite of this

number.

Exercise:

Problem:

(a) What is the acceleration due to gravity on the surface of the Moon?

(b) On the surface of Mars? The mass of Mars is 6.418×10^{23} kg and its radius is 3.38×10^{6} m.

Solution:

a) $1.62\ \text{m/s}^2$

b) $3.75\ \text{m/s}^2$

Exercise:

Problem:

(a) Calculate the acceleration due to gravity on the surface of the Sun.

(b) By what factor would your weight increase if you could stand on the Sun? (Never mind that you cannot.)

Exercise:

Problem:

The Moon and Earth rotate about their common center of mass, which is located about 4700 km from the center of Earth. (This is 1690 km below the surface.)

(a) Calculate the magnitude of the acceleration due to the Moon's gravity at that point.

(b) Calculate the magnitude of the centripetal acceleration of the center of Earth as it rotates about that point once each lunar month (about 27.3 d) and compare it with the acceleration found in part (a).

Comment on whether or not they are equal and why they should or should not be.

Solution:

a) $3.42 \times 10^{-5}\ \text{m/s}^2$

b) $3.34 \times 10^{-5}\ \text{m/s}^2$

The values are nearly identical. One would expect the gravitational force to be the same as the centripetal force at the core of the system.

Exercise:

Problem: Solve part (b) of [link] using $a_c = v^2/r$.

Exercise:

Problem:

Astrology, that unlikely and vague pseudoscience, makes much of the position of the planets at the moment of one's birth. The only known force a planet exerts on Earth is gravitational.

(a) Calculate the magnitude of the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.200 m away at birth (he is assisting, so he is close to the child).

(b) Calculate the magnitude of the force on the baby due to Jupiter if it is at its closest distance to Earth, some $6.29 \times 10^{11}\ \text{m}$ away. How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)

Solution:

a) $7.01 \times 10^{-7}\ \text{N}$

b) 1.35×10^{-6} N, 0.521

Exercise:

Problem:

The existence of the dwarf planet Pluto was proposed based on irregularities in Neptune's orbit. Pluto was subsequently discovered near its predicted position. But it now appears that the discovery was fortuitous, because Pluto is small and the irregularities in Neptune's orbit were not well known. To illustrate that Pluto has a minor effect on the orbit of Neptune compared with the closest planet to Neptune:

(a) Calculate the acceleration due to gravity at Neptune due to Pluto when they are 4.50×10^{12} m apart, as they are at present. The mass of Pluto is 1.4×10^{22} kg.

(b) Calculate the acceleration due to gravity at Neptune due to Uranus, presently about 2.50×10^{12} m apart, and compare it with that due to Pluto. The mass of Uranus is 8.62×10^{25} kg.

Exercise:

Problem:

(a) The Sun orbits the Milky Way galaxy once each $2.60 \text{ x } 10^{8}$ y, with a roughly circular orbit averaging $3.00 \text{ x } 10^{4}$ light years in radius. (A light year is the distance traveled by light in 1 y.) Calculate the centripetal acceleration of the Sun in its galactic orbit. Does your result support the contention that a nearly inertial frame of reference can be located at the Sun?

(b) Calculate the average speed of the Sun in its galactic orbit. Does the answer surprise you?

Solution:

a) 1.66×10^{-10} m/s^2

b) 2.17×10^{5} m/s

Exercise:

Problem: Unreasonable Result

A mountain 10.0 km from a person exerts a gravitational force on him equal to 2.00% of his weight.

(a) Calculate the mass of the mountain.

(b) Compare the mountain's mass with that of Earth.

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent? (Note that accurate gravitational measurements can easily detect the effect of nearby mountains and variations in local geology.)

Solution:

a) 2.937×10^{17} kg

b) 4.91×10^{-8}

of the Earth's mass.

c) The mass of the mountain and its fraction of the Earth's mass are too great.

d) The gravitational force assumed to be exerted by the mountain is too great.

Glossary

gravitational constant, *G*
: a proportionality factor used in the equation for Newton's universal law of gravitation; it is a universal constant—that is, it is thought to be the same everywhere in the universe

center of mass

the point where the entire mass of an object can be thought to be concentrated

microgravity

an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface

Newton’s universal law of gravitation

every particle in the universe attracts every other particle with a force along a line joining them; the force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

Satellites and Kepler’s Laws: An Argument for Simplicity

- State Kepler’s laws of planetary motion.
- Derive the third Kepler’s law for circular orbits.
- Discuss the Ptolemaic model of the universe.

Examples of gravitational orbits abound. Hundreds of artificial satellites orbit Earth together with thousands of pieces of debris. The Moon’s orbit about Earth has intrigued humans from time immemorial. The orbits of planets, asteroids, meteors, and comets about the Sun are no less interesting. If we look further, we see almost unimaginable numbers of stars, galaxies, and other celestial objects orbiting one another and interacting through gravity.

All these motions are governed by gravitational force, and it is possible to describe them to various degrees of precision. Precise descriptions of complex systems must be made with large computers. However, we can describe an important class of orbits without the use of computers, and we shall find it instructive to study them. These orbits have the following characteristics:

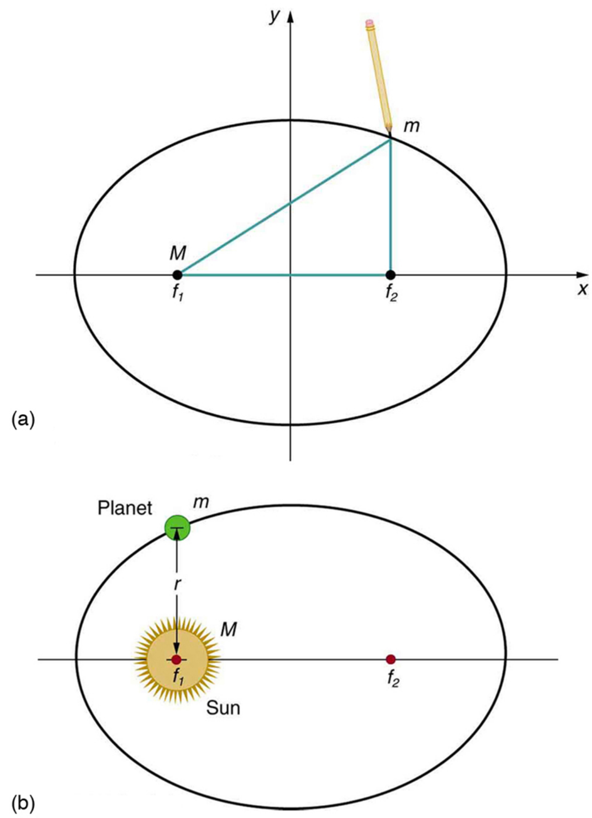
1. *A small mass m orbits a much larger mass M.* This allows us to view the motion as if M were stationary—in fact, as if from an inertial frame of reference placed on M —without significant error. Mass m is the satellite of M, if the orbit is gravitationally bound.
2. *The system is isolated from other masses*. This allows us to neglect any small effects due to outside masses.

The conditions are satisfied, to good approximation, by Earth’s satellites (including the Moon), by objects orbiting the Sun, and by the satellites of other planets. Historically, planets were studied first, and there is a classical set of three laws, called Kepler’s laws of planetary motion, that describe the orbits of all bodies satisfying the two previous conditions (not just planets in our solar system). These descriptive laws are named for the German astronomer Johannes Kepler (1571–1630), who devised them after careful study (over some 20 years) of a large amount of meticulously recorded observations of planetary motion done by Tycho Brahe (1546–1601). Such careful collection and detailed recording of methods and data are hallmarks of good science. Data constitute the evidence from which new interpretations and meanings can be constructed.

Kepler's Laws of Planetary Motion

Kepler's First Law

The orbit of each planet about the Sun is an ellipse with the Sun at one focus.



(a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci (f_1 and f_2) is a constant. You can draw an ellipse as shown by putting a pin at each focus, and then placing a string around a pencil and the pins and tracing a line on paper. A circle is a special case of an ellipse in which the

two foci coincide (thus any point on the circle is the same distance from the center). (b) For any closed gravitational orbit, m follows an elliptical path with M at one focus. Kepler's first law states this fact for planets orbiting the Sun.

Kepler's Second Law

Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times (see [link]).

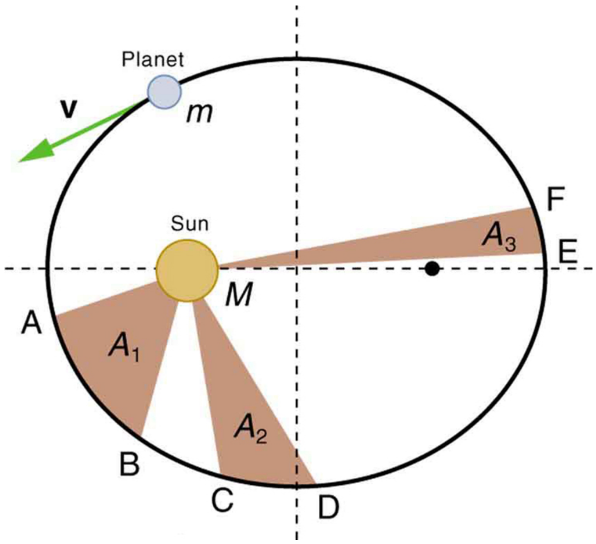
Kepler's Third Law

The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun. In equation form, this is

Equation:

$$\frac{{T_1}^2}{{T_2}^2} = \frac{{r_1}^3}{{r_2}^3},$$

where T is the period (time for one orbit) and r is the average radius. This equation is valid only for comparing two small masses orbiting the same large one. Most importantly, this is a descriptive equation only, giving no information as to the cause of the equality.



The shaded regions have equal areas. It takes equal times for m to go from A to B, from C to D, and from E to F. The mass m moves fastest when it is closest to M. Kepler's second law was originally devised for planets orbiting the Sun, but it has broader validity.

Note again that while, for historical reasons, Kepler's laws are stated for planets orbiting the Sun, they are actually valid for all bodies satisfying the two previously stated conditions.

Example:
Find the Time for One Orbit of an Earth Satellite
Given that the Moon orbits Earth each 27.3 d and that it is an average distance of 3.84×10^8 m from the center of Earth, calculate the period of an artificial satellite orbiting at an average altitude of 1500 km above Earth's surface.
Strategy

The period, or time for one orbit, is related to the radius of the orbit by Kepler's third law, given in mathematical form in $\frac{{T_1}^2}{{T_2}^2} = \frac{r_1^3}{r_2^3}$. Let us use the subscript 1 for the Moon and the subscript 2 for the satellite. We are asked to find T_2. The given information tells us that the orbital radius of the Moon is $r_1 = 3.84 \times 10^8$ m, and that the period of the Moon is $T_1 = 27.3$ d. The height of the artificial satellite above Earth's surface is given, and so we must add the radius of Earth (6380 km) to get $r_2 = (1500 + 6380)\ \text{km} = 7880\ \text{km}$. Now all quantities are known, and so T_2 can be found.

Solution

Kepler's third law is

Equation:

$$\frac{{T_1}^2}{{T_2}^2} = \frac{r_1^3}{r_2^3}.$$

To solve for T_2, we cross-multiply and take the square root, yielding

Equation:

$${T_2}^2 = {T_1}^2 \left(\frac{r_2}{r_1}\right)^3$$

Equation:

$$T_2 = T_1 \left(\frac{r_2}{r_1}\right)^{3/2}.$$

Substituting known values yields

Equation:

$$\begin{aligned} T_2 &= 27.3\ \text{d} \times \frac{24.0\ \text{h}}{\text{d}} \times \left(\frac{7880\ \text{km}}{3.84 \times 10^5\ \text{km}}\right)^{3/2} \\ &= 1.93\ \text{h}. \end{aligned}$$

Discussion This is a reasonable period for a satellite in a fairly low orbit. It is interesting that any satellite at this altitude will orbit in the same amount

of time. This fact is related to the condition that the satellite’s mass is small compared with that of Earth.

People immediately search for deeper meaning when broadly applicable laws, like Kepler’s, are discovered. It was Newton who took the next giant step when he proposed the law of universal gravitation. While Kepler was able to discover *what* was happening, Newton discovered that gravitational force was the cause.

Derivation of Kepler’s Third Law for Circular Orbits

We shall derive Kepler’s third law, starting with Newton’s laws of motion and his universal law of gravitation. The point is to demonstrate that the force of gravity is the cause for Kepler’s laws (although we will only derive the third one).

Let us consider a circular orbit of a small mass m around a large mass M, satisfying the two conditions stated at the beginning of this section. Gravity supplies the centripetal force to mass m. Starting with Newton’s second law applied to circular motion,
Equation:

$$F_{\text{net}} = ma_{\text{c}} = m\frac{v^2}{r}.$$

The net external force on mass m is gravity, and so we substitute the force of gravity for F_{net}:
Equation:

$$G\frac{\text{mM}}{r^2} = m\frac{v^2}{r}.$$

The mass m cancels, yielding
Equation:

$$G\frac{M}{r} = v^2.$$

The fact that m cancels out is another aspect of the oft-noted fact that at a given location all masses fall with the same acceleration. Here we see that at a given orbital radius r, all masses orbit at the same speed. (This was implied by the result of the preceding worked example.) Now, to get at Kepler's third law, we must get the period T into the equation. By definition, period T is the time for one complete orbit. Now the average speed v is the circumference divided by the period—that is,

Equation:

$$v = \frac{2\pi r}{T}.$$

Substituting this into the previous equation gives

Equation:

$$G\frac{\mathrm{M}}{r} = \frac{4\pi^2 r^2}{T^2}.$$

Solving for T^2 yields

Equation:

$$T^2 = \frac{4\pi^2}{\mathrm{GM}} r^3.$$

Using subscripts 1 and 2 to denote two different satellites, and taking the ratio of the last equation for satellite 1 to satellite 2 yields

Equation:

$$\frac{T_1^{\ 2}}{T_2^{\ 2}} = \frac{r_1^{\ 3}}{r_2^{\ 3}}.$$

This is Kepler's third law. Note that Kepler's third law is valid only for comparing satellites of the same parent body, because only then does the mass of the parent body M cancel.

Now consider what we get if we solve $T^2 = \frac{4\pi^2}{\text{GM}} r^3$ for the ratio r^3/T^2. We obtain a relationship that can be used to determine the mass M of a parent body from the orbits of its satellites:

Equation:

$$\frac{r^3}{T^2} = \frac{G}{4\pi^2} M.$$

If r and T are known for a satellite, then the mass M of the parent can be calculated. This principle has been used extensively to find the masses of heavenly bodies that have satellites. Furthermore, the ratio r^3/T^2 should be a constant for all satellites of the same parent body (because $r^3/T^2 = \text{GM}/4\pi^2$). (See [link]).

It is clear from [link] that the ratio of r^3/T^2 is constant, at least to the third digit, for all listed satellites of the Sun, and for those of Jupiter. Small variations in that ratio have two causes—uncertainties in the r and T data, and perturbations of the orbits due to other bodies. Interestingly, those perturbations can be—and have been—used to predict the location of new planets and moons. This is another verification of Newton's universal law of gravitation.

Note:
Making Connections
Newton's universal law of gravitation is modified by Einstein's general theory of relativity, as we shall see in Particle Physics. Newton's gravity is not seriously in error—it was and still is an extremely good approximation for most situations. Einstein's modification is most noticeable in extremely large gravitational fields, such as near black holes. However, general relativity also explains such phenomena as small but long-known deviations of the orbit of the planet Mercury from classical predictions.

The Case for Simplicity

The development of the universal law of gravitation by Newton played a pivotal role in the history of ideas. While it is beyond the scope of this text to cover that history in any detail, we note some important points. The definition of planet set in 2006 by the International Astronomical Union (IAU) states that in the solar system, a planet is a celestial body that:

1. is in orbit around the Sun,
2. has sufficient mass to assume hydrostatic equilibrium and
3. has cleared the neighborhood around its orbit.

A non-satellite body fulfilling only the first two of the above criteria is classified as “dwarf planet.”

In 2006, Pluto was demoted to a ‘dwarf planet’ after scientists revised their definition of what constitutes a “true” planet.

Parent	**Satellite**	**Average orbital radius *r*(km)**	**Period *T(y)***	**r^3 / T^2 (km^3 / y^2)**
Earth	Moon	3.84×10^5	0.07481	1.01×10^{19}
Sun	Mercury	5.79×10^7	0.2409	3.34×10^{24}

Parent	Satellite	Average orbital radius r(km)	Period $T(y)$	r^3 / T^2 (km^3 / y^2)
	Venus	1.082×10^8	0.6150	3.35×10^{24}
	Earth	1.496×10^8	1.000	3.35×10^{24}
	Mars	2.279×10^8	1.881	3.35×10^{24}
	Jupiter	7.783×10^8	11.86	3.35×10^{24}
	Saturn	1.427×10^9	29.46	3.35×10^{24}
	Neptune	4.497×10^9	164.8	3.35×10^{24}
	Pluto	5.90×10^9	248.3	3.33×10^{24}
Jupiter	Io	4.22×10^5	0.00485 (1.77 d)	3.19×10^{21}
	Europa	6.71×10^5	0.00972 (3.55 d)	3.20×10^{21}

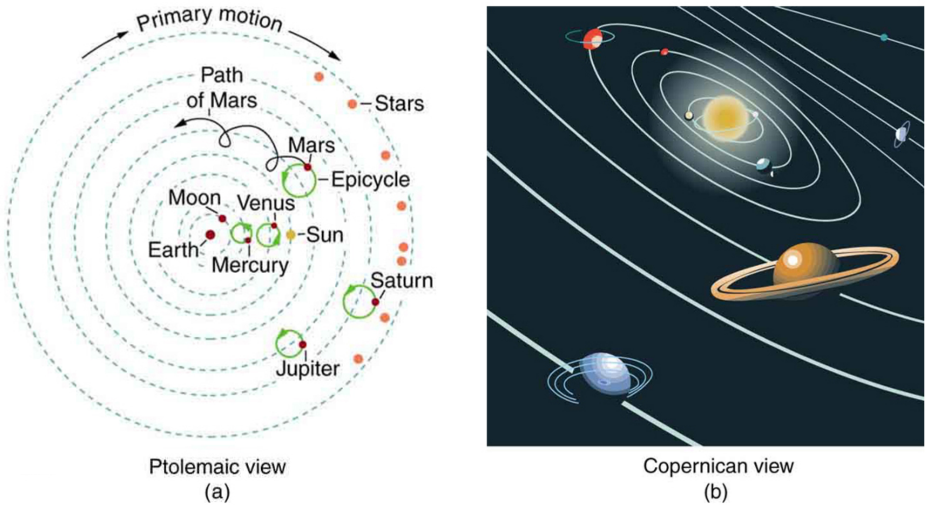
Parent	Satellite	Average orbital radius *r*(km)	Period *T(y)*	r^3 / T^2 (km^3 / y^2)
	Ganymede	1.07×10^6	0.0196 (7.16 d)	3.19×10^{21}
	Callisto	1.88×10^6	0.0457 (16.19 d)	3.20×10^{21}

Orbital Data and Kepler’s Third Law

The universal law of gravitation is a good example of a physical principle that is very broadly applicable. That single equation for the gravitational force describes all situations in which gravity acts. It gives a cause for a vast number of effects, such as the orbits of the planets and moons in the solar system. It epitomizes the underlying unity and simplicity of physics.

Before the discoveries of Kepler, Copernicus, Galileo, Newton, and others, the solar system was thought to revolve around Earth as shown in [link](a). This is called the Ptolemaic view, for the Greek philosopher who lived in the second century AD. This model is characterized by a list of facts for the motions of planets with no cause and effect explanation. There tended to be a different rule for each heavenly body and a general lack of simplicity.

[link](b) represents the modern or Copernican model. In this model, a small set of rules and a single underlying force explain not only all motions in the solar system, but all other situations involving gravity. The breadth and simplicity of the laws of physics are compelling. As our knowledge of nature has grown, the basic simplicity of its laws has become ever more evident.



(a) The Ptolemaic model of the universe has Earth at the center with the Moon, the planets, the Sun, and the stars revolving about it in complex superpositions of circular paths. This geocentric model, which can be made progressively more accurate by adding more circles, is purely descriptive, containing no hints as to what are the causes of these motions. (b) The Copernican model has the Sun at the center of the solar system. It is fully explained by a small number of laws of physics, including Newton's universal law of gravitation.

Section Summary

- Kepler's laws are stated for a small mass m orbiting a larger mass M in near-isolation. Kepler's laws of planetary motion are then as follows:

  Kepler's first law

  The orbit of each planet about the Sun is an ellipse with the Sun at one focus.

  Kepler's second law

Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times.

Kepler’s third law

The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun:
Equation:

$$\frac{{T_1}^2}{{T_2}^2} = \frac{{r_1}^3}{{r_2}^3},$$

where T is the period (time for one orbit) and r is the average radius of the orbit.

- The period and radius of a satellite’s orbit about a larger body M are related by
  Equation:

  $$T^2 = \frac{4\pi^2}{\text{GM}} r^3$$

  or
  Equation:

  $$\frac{r^3}{T^2} = \frac{G}{4\pi^2} M.$$

Conceptual Questions

Exercise:

Problem:

In what frame(s) of reference are Kepler’s laws valid? Are Kepler’s laws purely descriptive, or do they contain causal information?

Problem Exercises

Exercise:

Problem:

A geosynchronous Earth satellite is one that has an orbital period of precisely 1 day. Such orbits are useful for communication and weather observation because the satellite remains above the same point on Earth (provided it orbits in the equatorial plane in the same direction as Earth's rotation). Calculate the radius of such an orbit based on the data for the moon in [link].

Exercise:

Problem:

Calculate the mass of the Sun based on data for Earth's orbit and compare the value obtained with the Sun's actual mass.

Solution:

$1.98 \times 10^{30}\ \text{kg}$

Exercise:

Problem:

Find the mass of Jupiter based on data for the orbit of one of its moons, and compare your result with its actual mass.

Exercise:

Problem:

Find the ratio of the mass of Jupiter to that of Earth based on data in [link].

Solution:

$\frac{M_J}{M_E} = 316$

Exercise:

Problem:

Astronomical observations of our Milky Way galaxy indicate that it has a mass of about 8.0×10^{11} solar masses. A star orbiting on the galaxy's periphery is about 6.0×10^{4} light years from its center. (a) What should the orbital period of that star be? (b) If its period is 6.0×10^{7} years instead, what is the mass of the galaxy? Such calculations are used to imply the existence of "dark matter" in the universe and have indicated, for example, the existence of very massive black holes at the centers of some galaxies.

Exercise:

Problem: **Integrated Concepts**

Space debris left from old satellites and their launchers is becoming a hazard to other satellites. (a) Calculate the speed of a satellite in an orbit 900 km above Earth's surface. (b) Suppose a loose rivet is in an orbit of the same radius that intersects the satellite's orbit at an angle of 90º relative to Earth. What is the velocity of the rivet relative to the satellite just before striking it? (c) Given the rivet is 3.00 mm in size, how long will its collision with the satellite last? (d) If its mass is 0.500 g, what is the average force it exerts on the satellite? (e) How much energy in joules is generated by the collision? (The satellite's velocity does not change appreciably, because its mass is much greater than the rivet's.)

Solution:

a) $7.4 \times 10^{3}\ \mathrm{m/s}$

b) $1.05 \times 10^{3}\ \mathrm{m/s}$

c) $2.86 \times 10^{-7}\ \mathrm{s}$

d) $1.84 \times 10^{7}\ \mathrm{N}$

e) $2.76 \times 10^{4}\ \mathrm{J}$

Exercise:

Problem: **Unreasonable Results**

(a) Based on Kepler’s laws and information on the orbital characteristics of the Moon, calculate the orbital radius for an Earth satellite having a period of 1.00 h. (b) What is unreasonable about this result? (c) What is unreasonable or inconsistent about the premise of a 1.00 h orbit?

Solution:

a) 5.08×10^3 km

b) This radius is unreasonable because it is less than the radius of earth.

c) The premise of a one-hour orbit is inconsistent with the known radius of the earth.

Exercise:

Problem: **Construct Your Own Problem**

On February 14, 2000, the NEAR spacecraft was successfully inserted into orbit around Eros, becoming the first artificial satellite of an asteroid. Construct a problem in which you determine the orbital speed for a satellite near Eros. You will need to find the mass of the asteroid and consider such things as a safe distance for the orbit. Although Eros is not spherical, calculate the acceleration due to gravity on its surface at a point an average distance from its center of mass. Your instructor may also wish to have you calculate the escape velocity from this point on Eros.